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REPORT

RECOMMENDATIONS FOR EXISTING CONRETE DAMS

PROBABILISTIC ANALYSIS OF STABILITY

REPORT

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1 SUMMARY

The probabilistic analysis of the Reinoksvatn dam show that the dam has a sufficient safety, with a safety index higher than the target reliability index proposed in Probabilistic model code for concrete dams by M. W. Westberg and F. Johansson. [1]

> *Table 1-1: Minimum values for β in ultimate limit states. Reference period 1 year. [1]*

Dam consequence class	Minimum β minimum
A	5,2
B	4,8
C	4,2
U	3,8

The Reinoksvatn dam is classified as a class 2 dam [2], which corresponds to dam consequence class B in the table above.

Two loading situations has been considered; (i) winter season with ice loading and (ii) summer with floods. The probability distribution of the water level has been based on daily observations through 30 years combined with the design flood level , and then implemented according to seasonal variations for the two different load situations. The probability of overturning and sliding of the dam has been estimated for both load situations, as summarized below.

> *Table 1-2: Summary of results*

	Probability of failure	**B – safety index**
Winter - Overturning	$2.14 \cdot 10^{-7}$	5.05
Winter - Sliding	$5.17 \cdot 10^{-7}$	4.88
Summer - Overturning	$6.42 \cdot 10^{-8}$	5.28
Summer - Sliding	$2.93 \cdot 10^{-7}$	5.12

The probability of failure is calculated when the capacity in terms of overturning or sliding is exceeded. Capacity against overturning is assumed to be sufficient, when the resultant of all the forces exceed B/24, when measured from the downstream toe. Capacity against sliding, includes contributions from rock bolts and cohesion between the dam and the rock interface as well as the friction angle. The capacity against sliding is based on the Eurocode 2 formula on capacity in concrete joints. [3]

The probabilistic variable with the largest influence on the capacity is self-weight, water level and ice load.

2 INTRODUCTION

Requirements for stability of concrete dams in the current regulations are based on simplifications, which in many cases is very conservative. As a consequence unnecessary rehabilitation works may be carried out on dams that are safe, but does not meet the safety requirements. It is therefore desirable to look into how the assumptions for the calculations affect stability, in order to provide a better model to assess the actual safety and capacity. In addition, the calculations also indicate which elements that have most effect on the uncertainty when calculating sliding and overturning of the dam.

Dam structures must be checked for both overturning and sliding-stability. When calculating the sliding capacity, the current Norwegian regulations state that a plane interface between rock and the dam is to be assumed. The slope is determined by the difference in height between the dam heel and the dam toe. A friction angle of 45° (friction coefficient of 1.0) is assumed and cohesion is neglected. In addition, contributions from rock bolts are neglected, because of the uncertain capacity of their rock anchorage and the general condition of the bolts. Calculation of the sliding resistance require a safety factor of minimum 1.5, where the safety factor is given as the structural capacity divided by sum of horizontal forces acting on the structure.

When calculating the stability against overturning, the dam is assumed infinitely rigid. The resultant force is required to lie within the central dam foundation so that it can be assumed a pressure on the interface between the dam and the foundation.

The Reinoksvatn dam is used to illustrate the issues above with a probabilistic analysis.

The sliding capacity is based on the formula for design shear capacity for casting joints, according to Eurocode 2 [3]. This formula contains cohesion, friction and bolt capacity. This will make us capable to assess the contribution of other variables in addition to friction. Assuming a plane surface between the rock and the dam, makes the model simple and transparent while providing results that are directly comparable to the requirements of existing regulations.

In a probabilistic analysis, a safety factor is not relevant, instead a probability distributions for all variables is defined. Using statistical methods, one can thus calculate possible outcomes that combined with a failure criterion gives a probability of failure. This probability is then compared to a safety requirement. This method of analysis form the basis for the partial factors for materials and loads in eg. the Eurocodes.

This probabilistic analyses will be based on the "Probabilistic Model Code for Concrete Dams" written by Marie Westberg Wilde and Fredrik Johansson for Energiforsk in Sweden [1]. In addition, data from the Joint Committee on Structural Safety (JCSS) are used [4]. This committee is supported by six international associations in construction engineering – CEB, CIB, FIB, IABSE and RILEM.

3 REINOKSVATN DAM

Evaluation of the Reinoksvatn dam shows that the dam has insufficient sliding capacity when friction angle of 45° is assumed. A probabilistic analysis has been conducted to calculate the reliability of the dam, and to find the main uncertainties with respect to stability.

3.1 Background information

The Reinoksvatn dam is a concrete gravity dam located in Sørfold municipality in Norland County. The dam was built in the period 1985-86 and is connected to the Kobbelv hydro power plant.

> *Figure 3-1: Placement of Reinoksvatn dam [5]*

> *Figure 3-2: Airplane photo of the dam. The dam is 450 m long [5]*

> *Table 3-1: Reservoir information*

Magasin	
Maximum waterlevel (RWL):	680 m a.s.l.
Minimum waterlevel:	615 m a.s.l.
Precipitation field totalt:	48 km^2
Resevoir area	ca. 10 km^2

> *Table 3-2: Dam information*

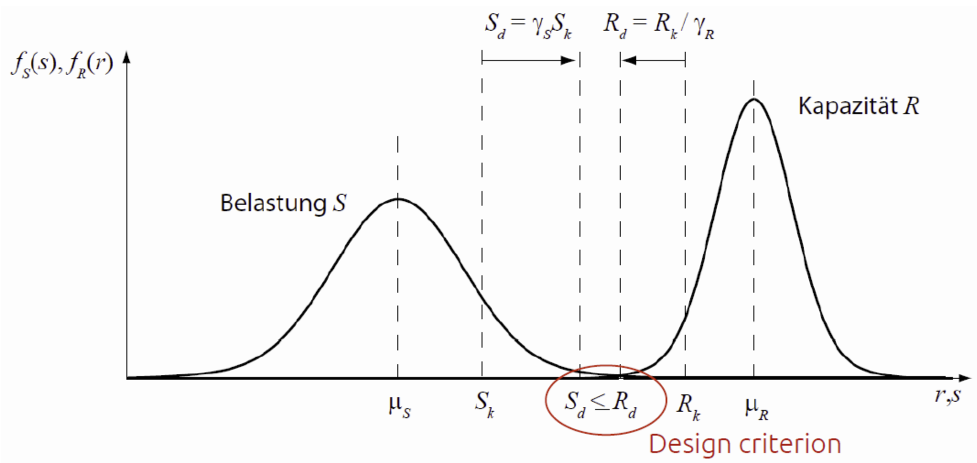
Dam class:	2
Dam type:	Gravity dam, concrete
Year of constrution:	1985-86
Dam height - max.	~21 m
Dam lenght	~450 m

4 PROBABILISTIC DESIGN

Uncertainty and randomness must be addressed in structural design. Uncertainty is present in loading, materials and models. The Norwegian regulations on dam safety (Damsikkerhetsforskriften) handle this by require a minimum safety against sliding and overturning. This method is called a deterministic method. A deterministic method is based upon empirical data and/or calibrations with probabilistic methods.

Deterministic methods work well for the type of structure/problem they have been calibrated for. The problem with deterministic methods is that they provide limited understanding of what causes uncertainties and how an increased reliability can be achieved. Deterministic methods are simple to apply in design, and therefore well suited for design of new structures. For reassessment of existing structures however, conservative assumptions can lead to extensive rehabilitations that are really not necessary.

In a probabilistic analysis, the uncertain variables are defined directly, and the probability of failure is calculated based on these variables. The method also returns how much each variable affect the reliability, and based on this it is possible to take effective actions to increase the reliability. These actions can range from doing measurements to get more accurate data (reducing uncertainty), to strengthening a specific component of the structure.



> *Figure 4-1: The overlapping region between load (Belastung) and resistance (Kapazität) define situation where the structure will fail [6]*

Figure 4-1 illustrate how both the loading on a structure, and the resistance of the structure is uncertain. A deterministic method uses a load factor (γ_S) and a material factor (γ_R) which increases the characteristic load, and decreases the characteristic capacity. If the design load is less than the design resistance, the structure is assumed to have sufficient safety. These factors are based on an acceptable level of risk. A probabilistic method calculates the probability of failure directly and compares that to the acceptable level of risk.

Eurocode 0 [7] allows the use of probabilistic design as an alternative to the use of partial safety factors. Section 3.5 (Limit state design):

(5) As an alternative, a design directly based on probabilistic methods may be used.

NOTE 1 The relevant authority can give specific conditions for use.

NOTE 2 For a basis of probabilistic methods, see Annex C.

4.1 FEM-model

The figure below show a 2D finite element model of a 1-meter wide section of the Reinoksvatn dam. The finite element analysis and probabilistic design is carried out with SOFiSTiK 2016.

A set of variables is used to define the uncertain variables of the dam. A statistical module of SOFiSTiK called RELY is used to define these variables and evaluate how they influence the safety of the dam. The variables are defined in section 4.2, and include:

- **Self weight**
- **Geometry**
- **Loading**

Two criterions are used to define failure of the dam. One criterion define sliding failure and the other define overturning. These are explained in detail in section 4.3.

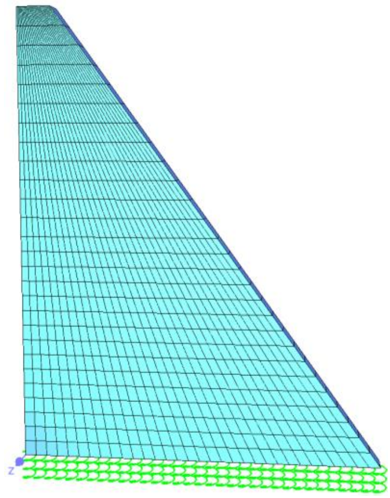
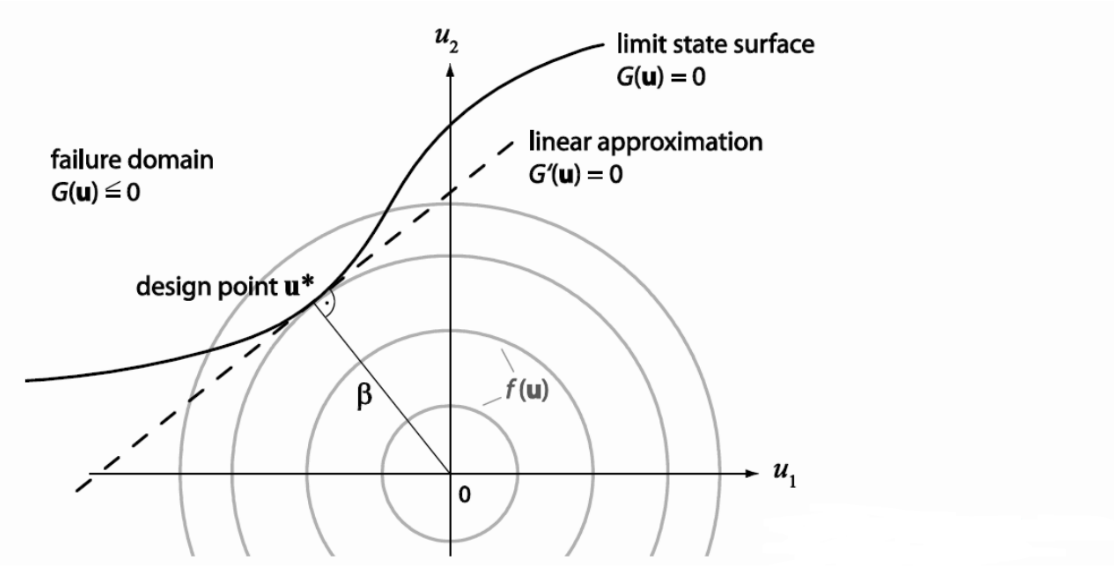


Figure 4-2: FEM-model in SOFiSTiK

The dam is modelled with 2D shell elements with a linear elastic material model. The interface towards the rock foundation is modelled with springs. Both a linear and a nonlinear model is used to model the concrete-rock interface. The nonlinear springs do not carry any tension loads, and lose their shear capacity when not in compression.

Two different approaches are used for evaluation of the probabilistic problem. A method called FORM (First Order Reliability Method) was primarily used to calculate the reliability. This method is based upon a linearization of the failure criterion, and is a very efficient way of calculating the reliability. In addition to returning the probability of failure for the structure, the method returns the sensitivities for each variable. But, as the method is based upon a linearization, it does not converge well for highly non-linear problems.

Figure 4-3 illustrate the FORM method. The circles illustrate the sample space for a probabilistic problem with 2 variables. The full drawn line defines the failure criterion (combinations of the variables leading to failure, e.g. sliding), and the dotted line illustrates a linearization of the failure criterion. The design point is the combination of variables causing failure which is most likely to occur. The β-value is the distance from the center (most probable outcome of the variables) to the design point. This value is directly related to the probability of failure. The vector from the center to the design point can be used to find the sensitivity of the variables. The value of each component of the vector reflects the sensitivity of that component. If we use Figure 4-3 as an example, we can see that variable u_1 and u_2 are about equally important. If the design point was placed on the u_1 axis, the reliability of the problem would be 100% dependent upon that variable.



> *Figure 4-3: A visualization of the FORM method.*

An alternative approach is to use a Monte Carlo simulation. This method is a level 3 method, or full probabilistic method, as it does not include any linearization. The method is considered exact and also works for highly non-linear problems, but is computationally costly. A large amount of realizations of the problem is carried out and the probability of failure is defined as the number of failed samples divided by the total number of samples.

A β-factor of 4.8 is suggested as a target reliability for class two dams (ref. section **Error! Reference source not found.**). This corresponds to a probability of failure of about $1{\cdot}10^{-6}$. The equation below estimates the coefficient of variation for a problem with probability of failure p_f and N number of samples.

$$\delta_{p_f} \approx \sqrt{\frac{1-p_f}{N \cdot p_f}}$$

The calculation below show the necessary number of simulations to get a coefficient of variation less than 10% for a probability of failure of 10^{-6}.

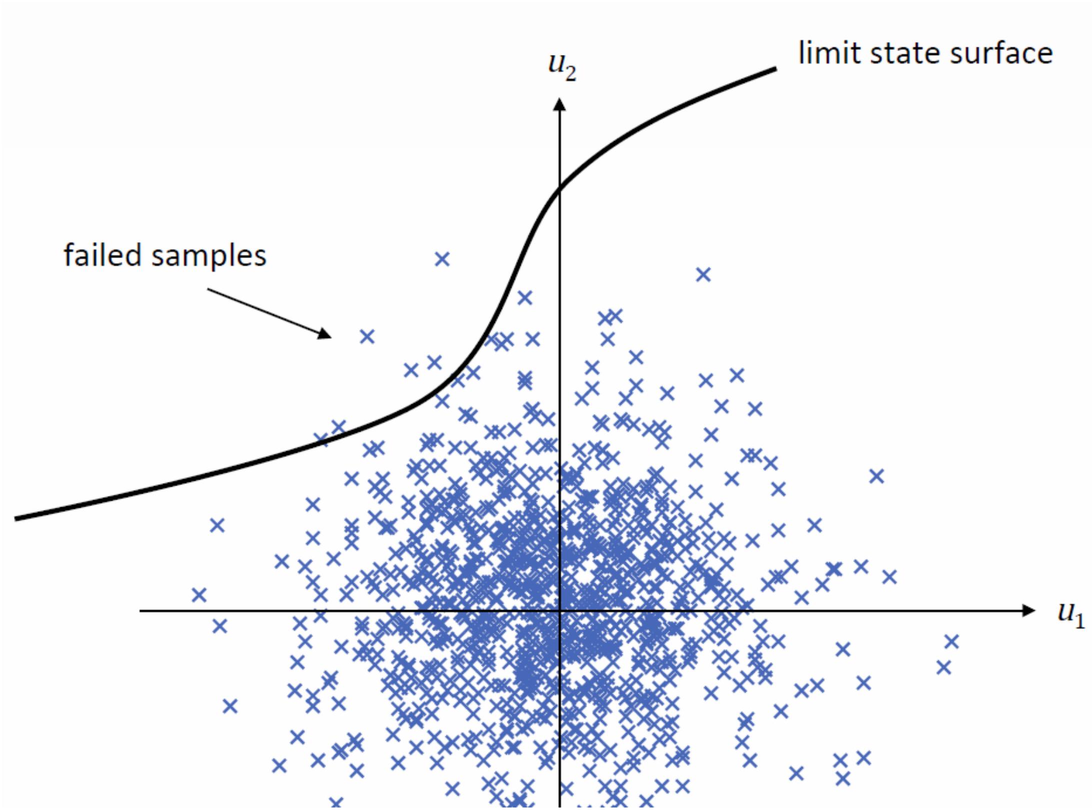
$$N = -\frac{p_f - 1}{p_f \cdot \delta^2} = \frac{10^{-6} - 1}{10^{-6} \cdot 0.1^2} = 10^8$$

If each simulation takes 1 second this results in a simulation time of 1160 days. If we allow a coefficient of variation of 50%, the necessary amount of simulations is 3 999 996, which is more manageable.

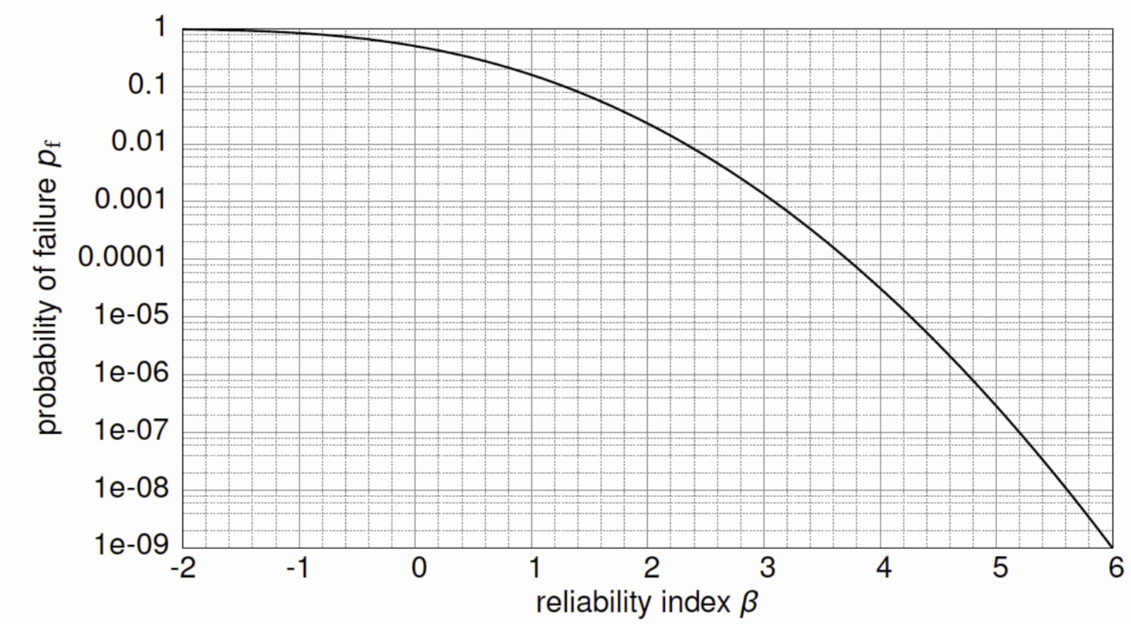
In this project, we have mainly used the FORM method, and then used Monte Carlo with a limited number of simulations to verify the results.

> *Table 4-1: Relation between reliability index and probability of failure (NS-EN 1990, annex C) [7]*

Pf	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1,28	2,32	3,09	3,72	4,27	4,75	5,20



> *Figure 4-4: Illustration of the Monte Carlo method for the same problem as in Figure 4-3.*



> *Figure 4-5: Relation between probability of failure and reliability index, β [8]*

4.2 Definition of variables

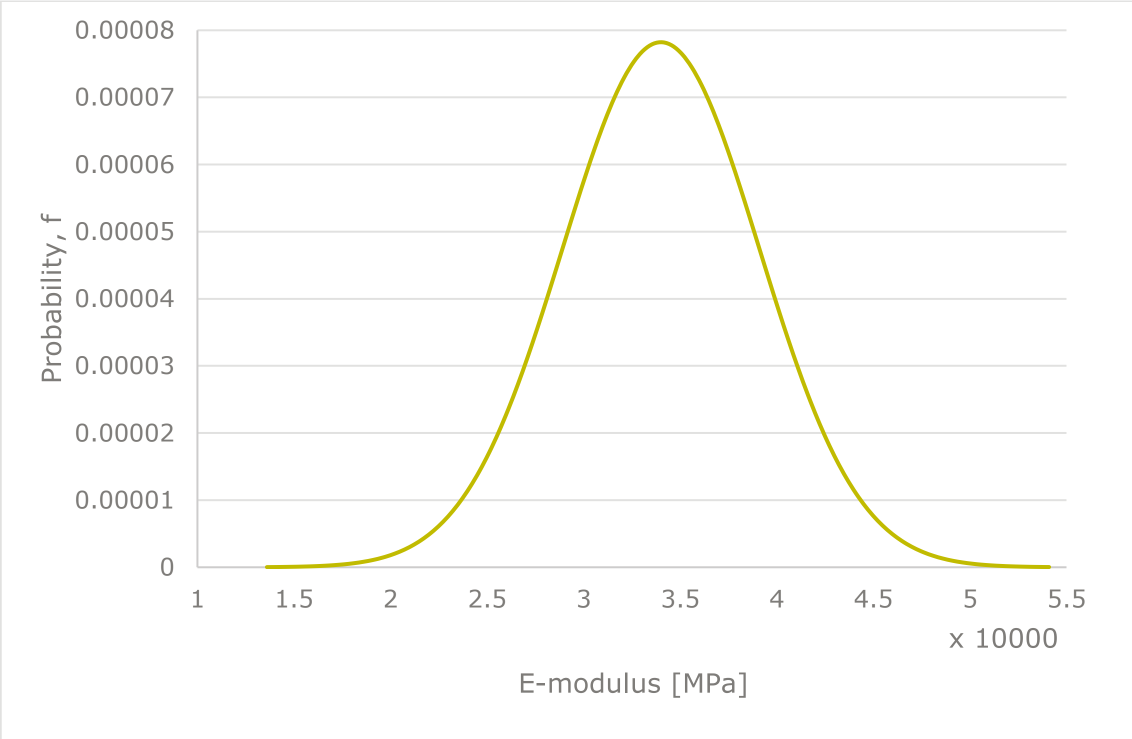
The variables describing the loads acting on the dam and the resistance of the structure is modeled with probability density functions (PDF). A common PDF for natural random variables is a normal distribution, described by a mean value and a standard deviation. Other distributions used in this project is log-normal distributions, which constrain the PDF to only positive values, and Gumbel distributions which models extreme values well.

The coefficient of variation is defined as the ratio of the standard deviation, σ, to the mean, μ.

$$c_v = \frac{\sigma}{\mu}$$

4.2.1 Stiffness of concrete

The E-modulus of the concrete is modelled with a normal distribution, with a mean value equal to the E-modulus of B35 (34 000 MPa) and a coefficient of variation (C.o.V) of 0.15. This C.o.V is recommended by the JCSS model code [4], table 3.1.1.

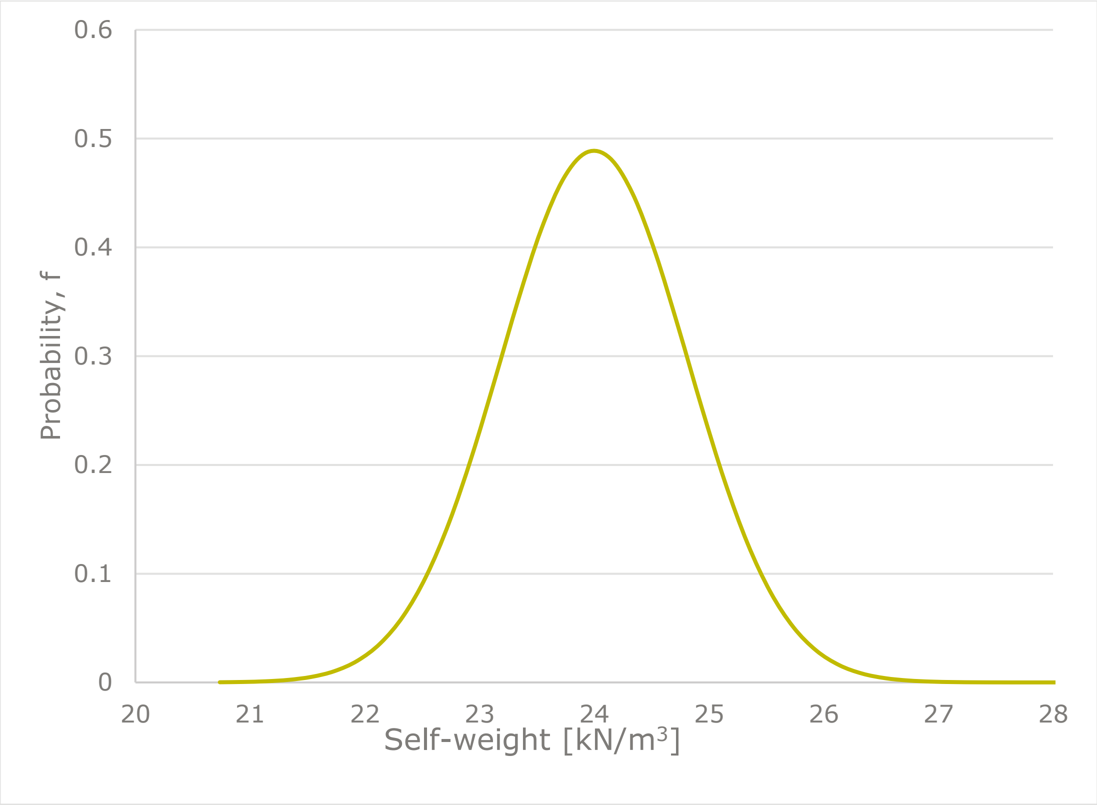


> *Figure 4-6: Probability density function for concrete E-modulus*

4.2.2 Structural self-weight

Structural self-weight is modelled with a normal distribution, with a mean value of 24 kN/m^3 and a coefficient of variation of 0.04. The C.o.V is recommended in table 2.1.1 in JCSS model code. [4]

In PMCCD chapter III:.1.4 (Probabilistic model code for concrete dams) a reduction of the C.o.V of 0.85 is proposed. [1] This gives a C.o.V of 0.034.

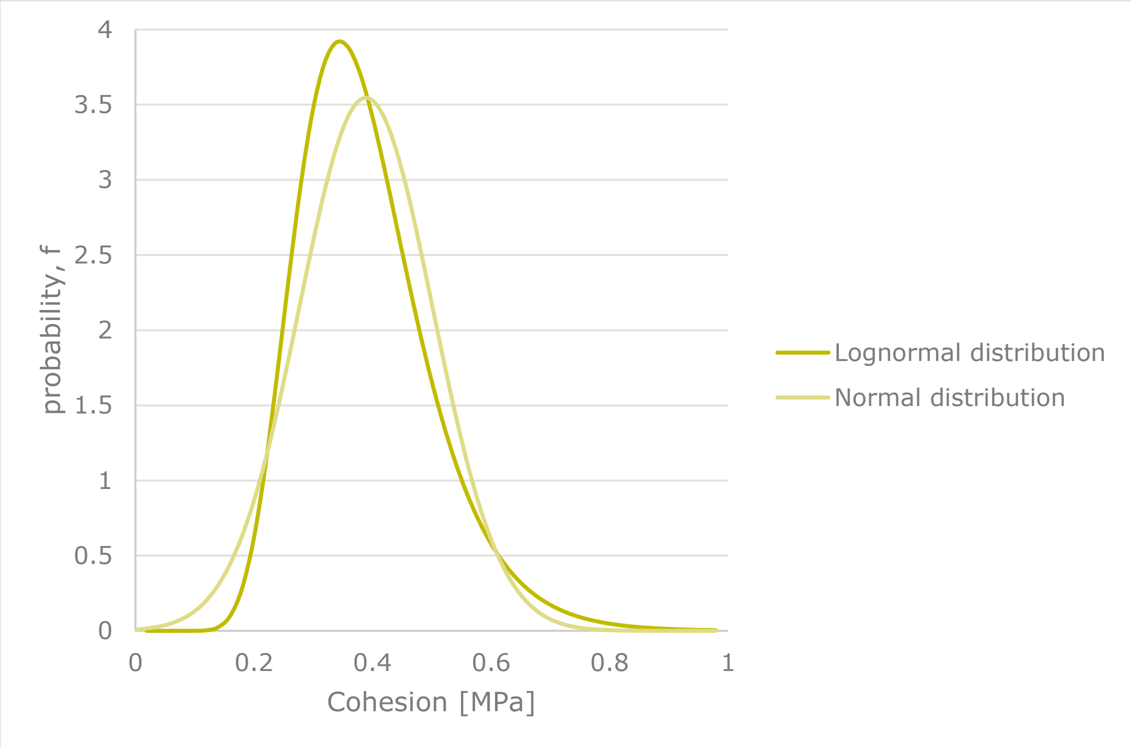


> *Figure 4-7: Probability density function for structural self-weight*

4.2.3 Cohesion

The PMCCD have no recommend values for cohesion. We have chosen to include this in our analysis to see what kind of effect cohesion has on the sliding capacity. The value is taken from tests carried out by NORUT (Northern Research Institute) at the Målset dam [9]. The probability density function is implemented in the analysis as a log-normal distribution with a mean value of 0.389 MPa and a coefficient of variation of 0.289.

The figure below shows a normal distribution, which was first implemented with the same values. This gave some problems due to the possibility of negative values for the cohesion, which is impossible to achieve in reality. A log-normal distribution was therefore considered to provide the best fit to the experimental data. This distribution is considered conservative as it is shifted more towards lower values of cohesion.



> *Figure 4-8: Probability density function, cohesion*

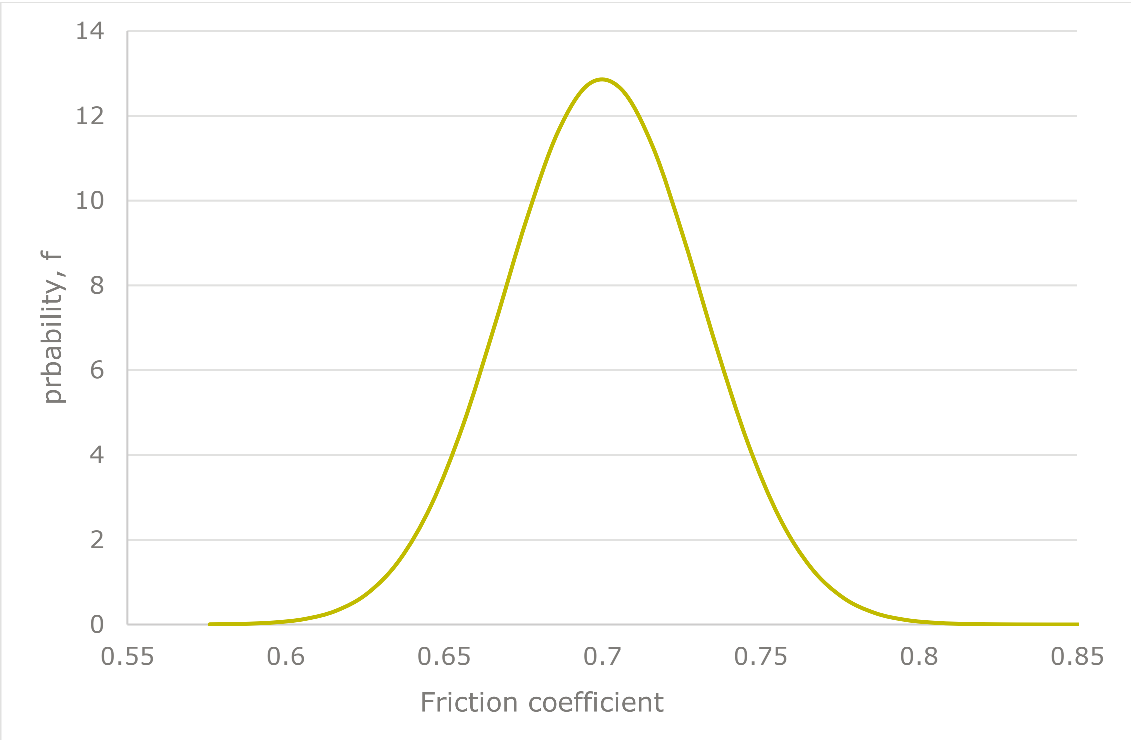
Eurocode 2 recommends a cohesion of $0.2 \cdot f_{ctd}$ for a smooth surface between concrete cast at different times. This would give a cohesion of 0.328 MPa for B30 concrete. A smooth surface is defined as a slipformed or extruded surface, or a surface left without further treatment after vibration. [3] This value is similar to the mean value proposed by NORUT.

Cohesion is only included in estimation of sliding capacity. The cohesion does not contribute when calculating stability against overturning.

4.2.4 Friction

The friction angle is included according to chapter III:.3 in PCCMD, with a tan φ-mean value of 35° (0,7) and a standard deviation of 1,75° (0,031). [1]

The interface between rock and concrete is considered to be plane. Any frictional resistance resulting from macro-roughness is therefore not considered. Such resistance would probably lead to a shear failure in the concrete and not sliding of the dam structure.



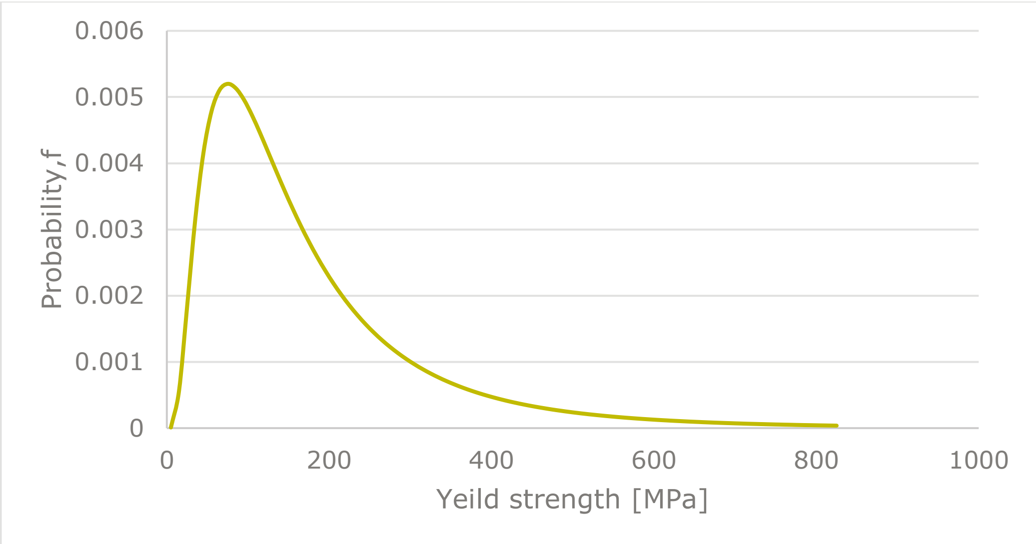
> *Figure 4-9: Probability density function, friction*

Eurocode 2 proposes a friction factor of 0.6 for a smooth surface between concrete cast at different times. This is somewhat lower than the mean value applied in the analysis, but the surface between rock and concrete is rougher than the smooth surface, defined in EC2. [3]

Friction factor is only included in estimation of sliding capacity. The friction factor does not contribute when calculating stability against overturning.

4.2.5 Rock bolts, Yield strength of reinforcement

The yield strength of rock bolts is modelled with a lognormal probability function. We have included this in the analysis only on the capacity side for sliding according to the shear formula in EC2. The mean value is 180 MPa, with a coefficient of variation of 0.89. This variable primarily accounts for the uncertain anchorage of the bolts, as the variation in actual yield strength of steel is very small. Tests carried out on existing structures have actually resulted in fracture of the steel, and not the bolt-rock interface, which implies that this distribution is very conservative. [10]



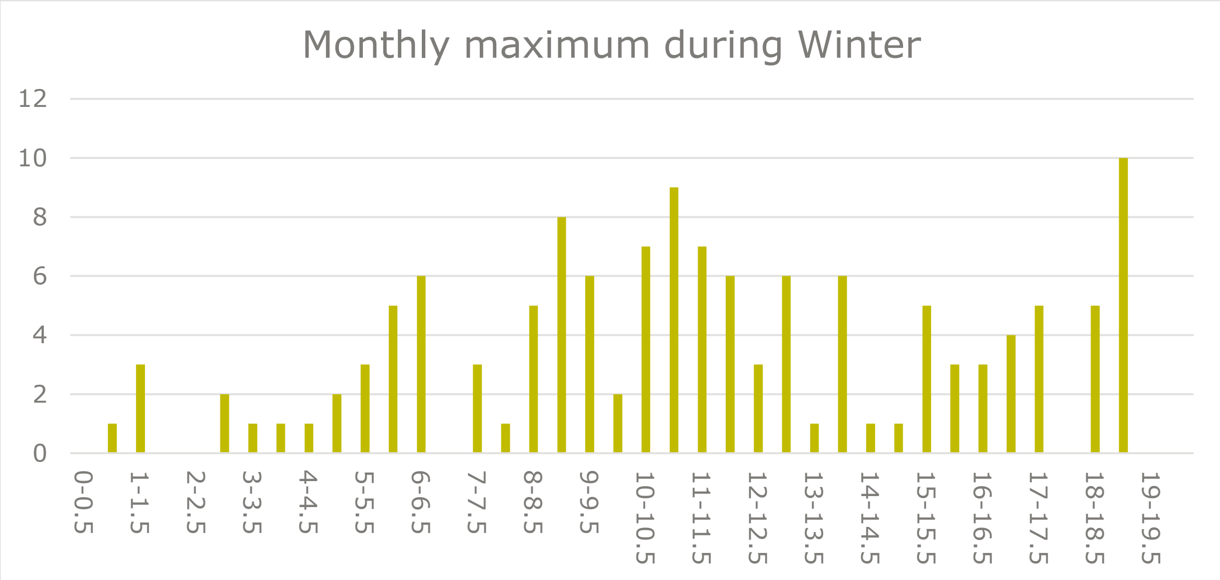
> *Figure 4-10: Probability density function, yield strength of reinforcement*

Rock bolts are only included in estimation of sliding capacity, as we use the shear formula in EC2. This capacity will not be relevant for calculation of overturning.

4.2.6 Hydrostatic pressure

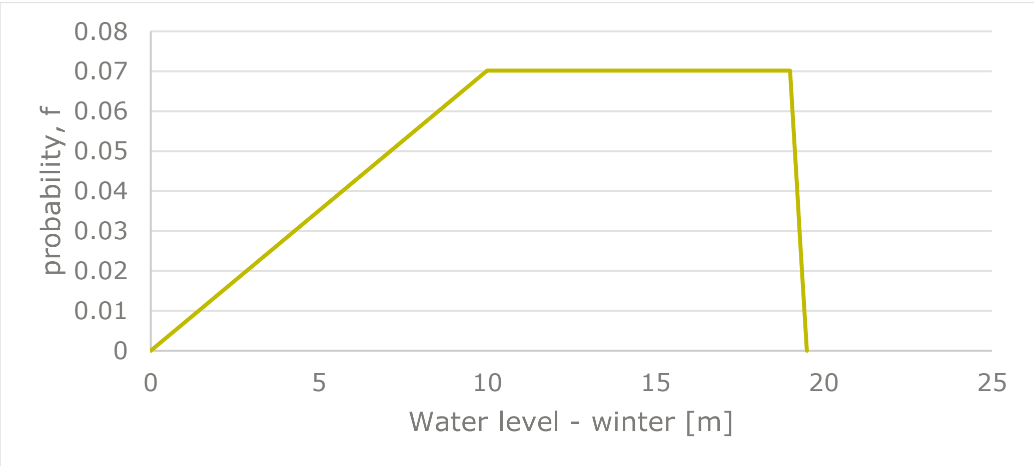
The probability density functions for the water level is based on measurements of the dam water level from 1988 to 2017. In the analysis we investigate two different load cases. One for water level with ice loading and one with a flooding situation.

For the water level with ice loading, the measurement during the winter season is used. This is due to the fact that maximum ice-load does not occur during the summer. The winter season is defined to be from November through May. The figure below shows a histogram of the monthly maximum water levels during winter. As seen in the histogram, the water level does not exceed highest operating water level (HRV) during the winter season. There are also 49 months in which the water level is below the foot of the dam. These were omitted in the graph.



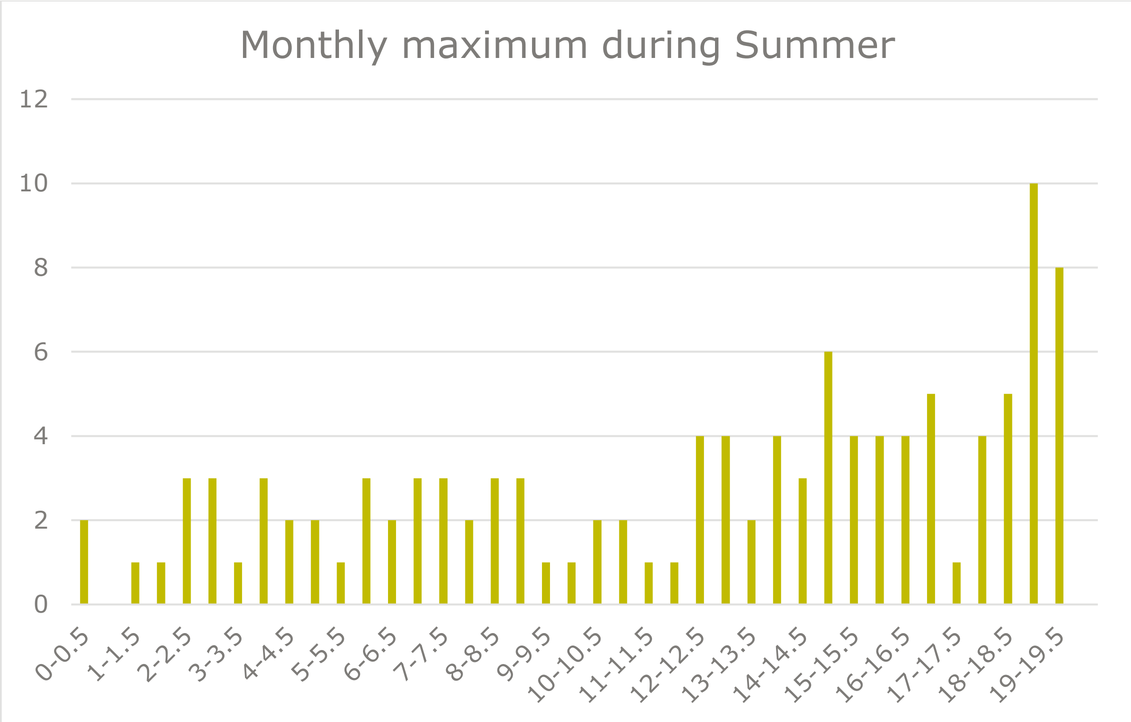
> *Figure 4-11: Monthly maximum water level, winter*

Figure 4-12 shows the probability density function based on the histogram above, where the maximum water level is set to 19.5 m, 0.5 m above highest operating water level (HRV). The probability of the water exceeding 19.0 m is 1.7 % for this distribution. None of the measurements during the winter season exceeded HRV.



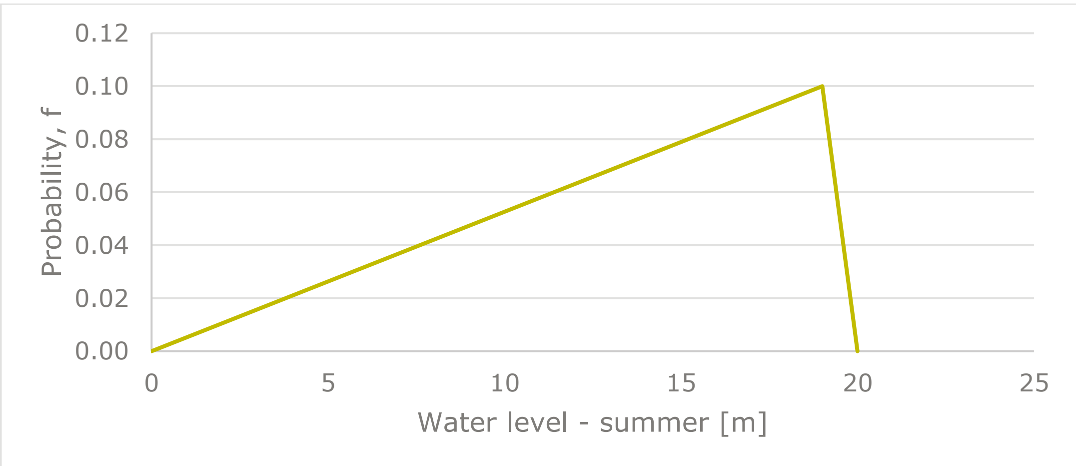
> *Figure 4-12: Probability density function, water level, ice load case*

For the flood situation, we have used measurements of the water level from the summer season. The summer season is from June through October. The figure below shows a histogram of the monthly maximum for the Reinoksvatn. As can be seen from the figure, the dam does not have any water levels measured above 19.5 m. There are also 20 months in which the water level is below the dam heel. These observations were omitted from the graph and not included in the calculations.



> *Figure 4-13: Monthly maximum water level, summer*

Figure 4-14 shows the probability density function for the flood case, based on the histogram above. The maximum water level is set to 20 m (dam height). This density function gives probability of the water level exceeding 19.8 (1.5·Qdim) of 0.2%. The water level has never exceeded 19.8 meter during the period.



> *Figure 4-14: Probability density function, water level, flood case*

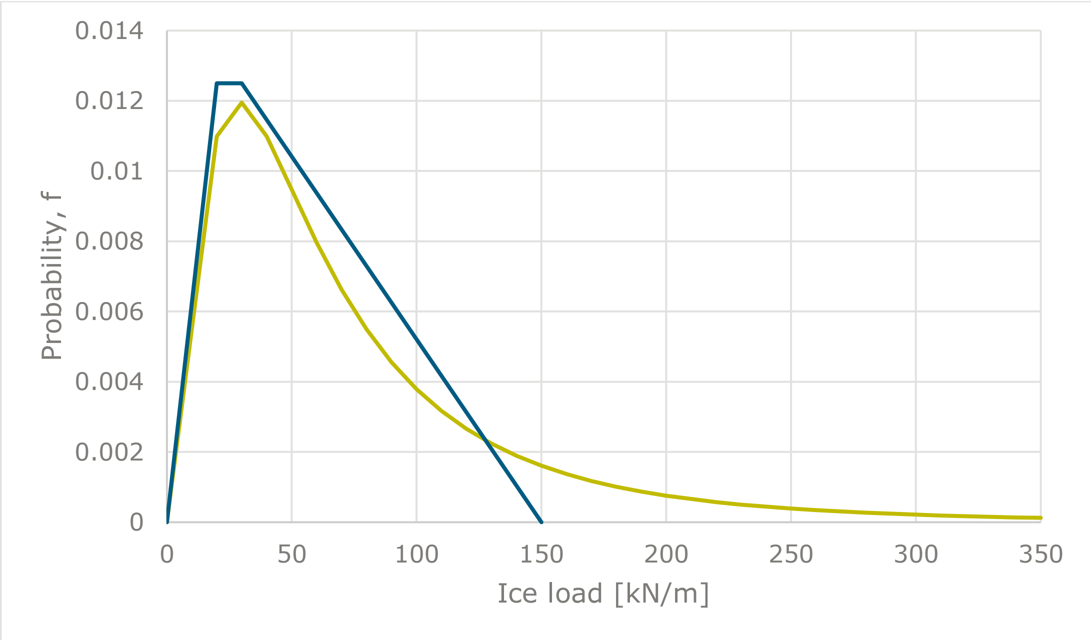
In probabilistic analysis, the yearly probability of the variable is used as reference. We chose to use the monthly maximum values to base our probability densities on, due to larger data sampling.

The uplift pressure is implemented in the calculations according to normal practice. It varies linearly from the heel to the toe of the dam if the resultant is within 1/3 of the base width. If the resultant is downstream this area, the uplift pressure is constant over the area without compression.

4.2.7 Ice loading

The ice load according to PMCCD, should be described by a log-normal distribution, shown in the figure below. According to NVE guidelines, an alternative method for calculating ice load returns a load of 125 kN/m for Sørfold municipality. This is based on a frost level with a return period of 100 years.

Initially, the analysis was executed with the log-normal distribution proposed in PMCCD which resulted in unrealistic high ice load over 800 kN/m, as the log-normal distribution ranges from 0 to ∞. As the ice load is in reality a deformation load, we implemented a trapezoidal distribution to fit the log-normal curve but not exceed 150 kN/m. This distribution gives a 3% annual probability of the ice load to exceed 125 kN/m.



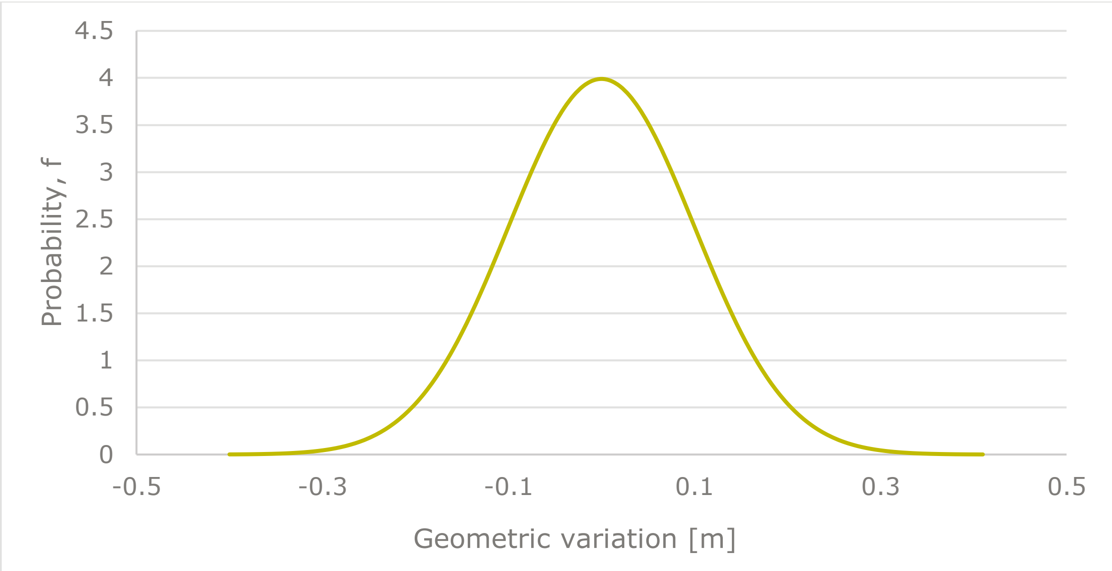
> *Figure 4-15: Probability density function, ice load [1]*

The ice load is applied 0.25 meters below the water level, e.g. the ice thickness is assumed 0.5 meter with the resultant acting at the center of the layer of ice.

4.2.8 Geometric variation

The geometric variation is included as a delta applied to the height and width of the dam. This probability density function is based on normal building tolerances, to investigate what kind of impact this can have on failure of the dam.

The probability density function is described with a normal distribution, with a mean value of 0 and a standard deviation of 0.1m.



> *Figure 4-16: Probability density function, geometric variation*

4.3 Failure criterion

A failure criterion has to be defined for each failure mode. Failure happens when the failure criterion is less than, or equal to, zero. To achieve convergence with FORM (section 4.1) the failure criterion must be expressed so that the software also know how close it is to failure. A normal expression is:

$$R - F > 0$$

Where:

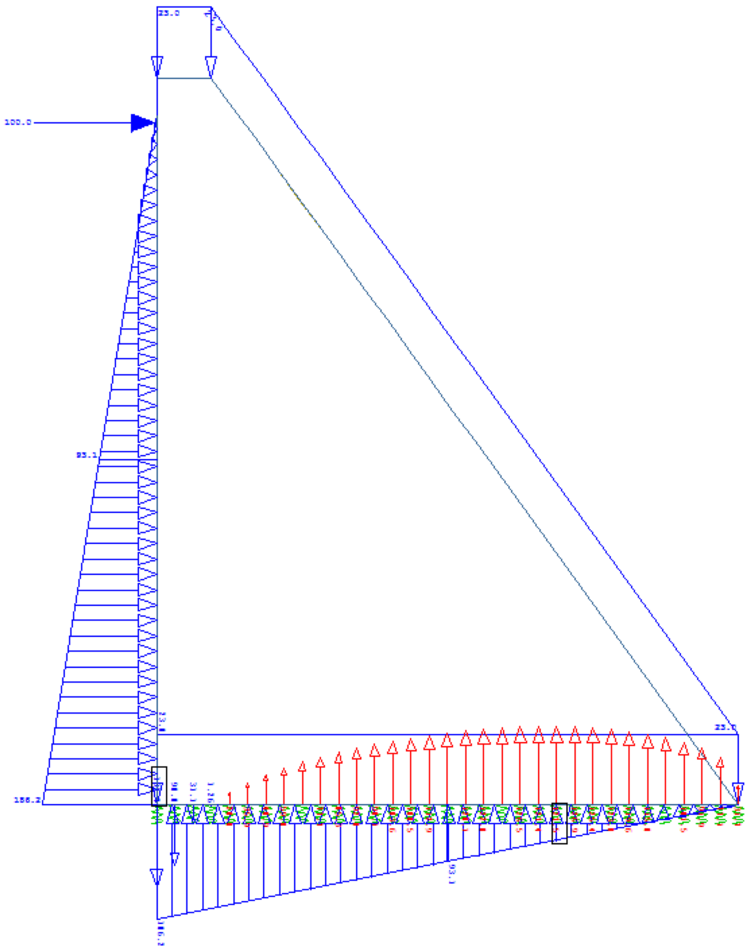
R is resistance
F is loading

4.3.1 Overturning

The failure criterion is specified as $e_{Ed} - e_{Rd} > 0$

The eccentricity of the resultant from the dam toe, e_{Ed}, is calculated by reading the spring forces under the dam and finding the resultant placement based on the pressure distribution of these.

Overturning occurs when the resultant is downstream of the dam toe (i.e. e_{Ed}<0). In reality crushing of the concrete at the dam toe will occur before this, so the design value is set to e_{RD} = B/24 (≈0.67m).



> *Figure 4-17: Pressure on the base of the dam for regulation water level and normal ice load*

4.3.2 Sliding

The failure criterion is specified as $V_{Rd} - V_{Ed} > 0$

The sliding force, V_{Ed}, is the sum of all horizontal forces.

The design sliding resistance, V_{Rd}, is taken as:

$$V_{Rd} = N' \cdot \mu + A_c \cdot c + \mu \cdot f_y \cdot A_s$$

Where;

- **N' = the sum of all vertical forces**
- **μ = the friction coefficient**
- **A_c = the area of the foundation in pressure**
- **c = the cohesion**
- **f_y = yield strength of rock bolts**
- **A_a = the area of the rock bolts**

Rock bolts are only included in estimation of sliding capacity, as we use the shear formula in EC2. This capacity will not be relevant for calculation of overturning.

4.4 Results

The results of the probabilistic analysis gives us a β, which should exceed the target safety index for the dam consequence class, given in table PI-6.2 in PMCCD. [1]

It also returns the design point, which is the combination of variables leading to failure that is most likely to occur, shown as “Equivalent x” in the tables below.

The alpha values reflect the sensitivity of each variable and reflect how important that variable is compared to the other variables. To improve the safety of the dam the most sensitive variable should be addressed first.

The results for overturning and sliding are presented below for two different situations.

Situation 1: Winter season with the water level as shown in Figure 4-12 and ice load according to 4.2.7

Situation 2: Summer season with the water level as shown in Figure 4-13 and no ice loading

Rock bolts, cohesion and the friction factor is only included in estimation of sliding capacity. These variables does not contribute to capacity against overturning.

4.4.1 Situation 1: Winter season

Failure mode	Probability of failure	β – safety index
Overturning	**$2.14 \cdot 10^{-7}$**	**5.05**
Sliding	**$5.17 \cdot 10^{-7}$**	**4.88**

Variable	Overturning		Sliding	
	Equivalent x	Alpha	Equivalent x	Alpha
E-modulus [MPa]	34 000	0 %	34 000	0 %
Ice load [kN/m]	120.2	11 %	119.76	12 %
Self-weight [kN/m^3]	20.90	56 %	21.10	53 %
Rock bolts [kPa]	Not included		129 358	0 %
Water level [m]	19.31	30 %	19.30	32 %
Delta height [m]	-0.06	1.5 %	-0.05	1.0 %
Delta width [m]	-0.03	0.4 %	-0.04	0.8 %
Cohesion [MPa]	Not relevant		0.32	1.1 %
Friction	Not relevant		0.70	0 %

4.4.2 Situation 2: Summer season

Failure mode	Probability of failure	β– safety index
Overturning	**$6.42 \cdot 10^{-8}$**	**5.28**
Sliding	**$2.93 \cdot 10^{-7}$**	**5.12**

Variable	Overturning		Sliding	
	Equivalent x	Alpha	Equivalent x	Alpha
E-modulus [MPa]	34 000	0 %	34 000	0 %
Self-weight [kN/m^3]	20.5	66 %	20.67	63 %
Rock bolts [kPa]	Not included		129 897	0 %
Water level [m]	19.83	32 %	19.83	34 %
Delta height [m]	-0.07	1.7 %	-0.06	1.2 %
Delta width [m]	-0.03	0.4 %	-0.05	0.8 %
Cohesion [MPa]	Not relevant		0.32	0.9 %
Friction	Not relevant		0.70	0 %

4.4.3 Discussion of results

The reliability index ranges from 4.88-5.28 for the four design cases. In general, overturning has a higher safety factor than sliding, which corresponds to the results from calculation of the stability in accordance with the traditional method in the dam safety regulations.

According to PMCCD a target reliability index above 4.8 is suggested for dam consequence class B. The Reinoksvatn dam is classified as a class 2 dam and should be compared to class B in the table below.

> *Table 4-2: Minimum values for β in ultimate limit states. Reference period 1 year. [1]*

Dam consequence class	Minimum β minimum
A	5,2
B	4,8
C	4,2
U	3,8

> *Table 4-3: Dam consequence classes [1]*

Dam consequence class	Consequences	
A	May cause loss of many human lives	Failure may lead to a crisis affecting many people and large parts of the society and threaten fundamental values and functions.
B	May cause loss of human lives	Failure may lead to large regional and local consequences and disturbances
C	Negligible risk of loss of human lives	
U	Without dam consequence class.	

> *Table 4-4: Classification criteria according to Damsikkerhetsforskriften [2]*

Konsekvens-klasse	Boenheter	Infrastruktur, samfunnsfunksjoner	Miljø og eiendom
4	> 150		
3	21-150	Skade på sterkt trafikkert veg eller jernbane, eller annen infrastruktur, med spesielt stor betydning for liv og helse	Stor skade på spesielt viktige miljøverdier eller spesielt stor skade på fremmed eiendom
2	1 - 20	Skader på middels trafikkert veg eller jernbane eller annen infrastruktur med stor betydning for liv og helse.	Stor skade på viktige miljøverdier eller stor skade på fremmed eiendom
1	Midlertidig oppholdssted tilsvarende < 1 permanent boenhet	Skader på mindre trafikkert veg eller annen infrastruktur med betydning for liv og helse	Skade på miljøverdier eller fremmed eiendom

The probabilistic analysis show that the dam has a sufficient reliability index within the defined probabilities in chapter 4.2.

As seen by the alpha values in all load cases, the self-weight of the structure is the largest factor of uncertainty in the model. This is uncertainty can be reduced greatly by taking concrete samples and measuring the self-weight. We do not have access to any measurements of the density of the concrete in the dam. The distribution and mean for the self-weight is therefore based on typical values in a design phase. By measuring the concrete weight, the uncertainty can be reduced and the safety index may increase.

The second most significant variable is the water level. The used distributions are roughly estimated based on observations. A more detailed evaluation of the data could give better statistical basis to apply the corresponding flood water level distribution.

The winter season analysis gives highest probability for failure, due to ice load. In reality, the ice load is a deformation load that will disappear even with very small deflections. In addition, a flood water level of 0.5 m above maximum normal operating level (HRV) is possible during the winter season. Restricting this variable can reduce the probability of failure.

The small alpha values for the cohesion, only 1%, may seem surprising. Cohesion is dependent on the area in compression in the failure plane. When, e.g. reducing the self-weight, the area in compression may also reduce. This effect has much greater influence on the results than the value of cohesion itself. This can explain the low alpha value for cohesion.

The alpha value of rock bolts is zero. With a mean value for the yield-strength at 180 MPa and only one ø25 mm bolt per meter along the dam axis, the capacity of these are very small and corresponding contribution to total sliding capacity is negligible.

To increase the reliability of the dam, further investigations of the following factors are recommended:

- **Self-weight:** Carrying out tests of the concrete density could reduce the variance of the probability density function, and lead to better reliability.
- **Ice-pressure measurements:** As the ice load is a deformation load and is in this analysis included as a static load, measurements of the ice-pressure on the dam could give a better indication of the ice load value.
- **Ice-pressure effect**: Implementing the ice load as a small deformation in the analysis. Further investigations and modeling of how the ice pressure affect the dam can also influence the results.
- **Flood levels:** New flood calculations that includes the real statistical distribution of flood events would improve the results. The present flood calculations are based on assumptions regarding the initial water level and do not reflect the actual probability.

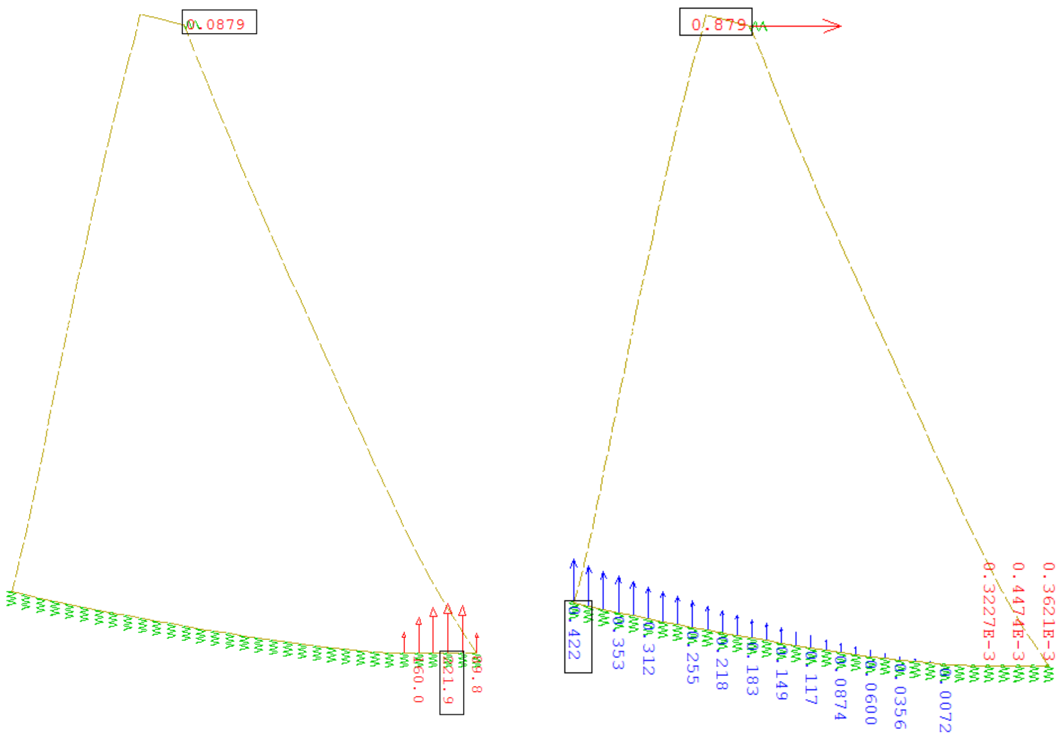
5 VERIFICATION OF RESULTS

The four cases with their corresponding failure load is analyzed with non-linear springs under the foundation to verify that the linear springs used in the probabilistic analysis is valid.

In the probabilistic analysis the springs under the foundation is modeled as linear. Due to the huge number of analysis need to perform the probabilistic analysis, the linear spring reduces the time for analyzing and ensures convergence of the β–safety factor.

The non-linear spring model include non-tension springs along the foundation that has no stiffness in tension. An extra spring, with very little stiffness has been included, at the top of the dam, in order to register the deformations at the top of the dam and prevent an unstable analysis. The spring stiffness is very small for the first 30 mm of deformation and then gradually increased for larger deformations, to secure convergence of the non-linear analysis.

The figure below show the forces and deformations in the springs for the failure point for sliding with ice load.



> *Figure 5-1: Deformation and spring forces for non-linear analysis.*

The standard load cases, used to determine the capacity according to regulations, has been analyzed with the non-linear model. The results are presented in the Table 5-1.

> *Table 5-1: Dam capacity without cohesion*

	Regulation water level (19 m) +ice load (100 kN/m)	Design flood (19.69 m)	Accidental flood (19.8 m)
Eccentricity from toe [m]	5.44	5.57	4.81
Eccentricity requirement [m]	5.33	5.33	2.67
Horizontal forces [kN]	1 869	1 900	1 921
Sliding capacity, with friction coefficient = 1.0 [kN]	2 535	2 481	2 474
Factor of safety, against sliding	1.36	1.31	1.29
Sliding requirement	1.5	1.5	1.1

As seen in the table above, the dam has insufficient resistance against sliding for ice load and the design flood situation.

Table 5-2 shows the sliding capacity with cohesion and rock bolts, here using the mean values used in the probabilistic analysis. 0.389 MPa for the cohesion, 180 N/mm^2 steel capacity for rock bolts and a friction coefficient of 0.7. As seen in the table, this gives a significantly higher capacity than just using a friction coefficient.

The rock bolts contributes only to a small increase in the capacity of 62 kN, and has no effect on the stability. In comparison, the cohesion contributes with approximately 6 000 kN increase in capacity for the standard load cases. Even a small cohesion value of 0.1 MPa could give an increase in capacity of 1 500 kN, compared to a situation when cohesion is not considered.

> *Table 5-2: Dam sliding capacity, with cohesion and rock bolts*

	Regulation water level (19 m) +ice load (100 kN/m)	Design flood (19.69 m)	Accidental flood (19.8 m)
Sliding capacity, with cohesion and rock bolts [kN]	8 061	8 023	8 017
Factor of safety, against sliding	4.31	4.22	4.17
Sliding requirement	1.5	1.5	1.1

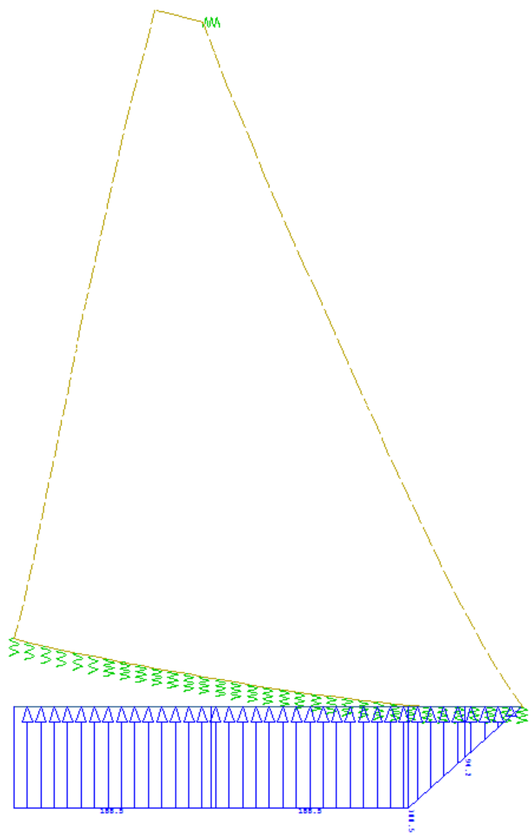
The design points that results in failure from the probabilistic analysis has been analyzed in the non-linear model. As seen in the table below, the non-linear model confirms that the design points are approximately the same using linear springs.

> *Table 5-3: Verification of model*

	Situation 1: Winter season	Situation 2: Summer season
Eccentricity from toe [m]	0.68	0.65
Failure criterion in probabilistic model [m]	0.67	0.67
Horizontal forces [kN]	1 924	1 913
Sliding capacity	1 924	1 945

The assumption of that the foundation is in compression, when the resultant is upstream 1/3 of the base width is supported by the non-linear analysis.

The pore pressure distribution under the foundation is assumed similar in the non-linear analysis as in the probabilistic. The figure below shows the applied pore pressure on the deformed structure (deformation is enlarged by 5000).



> *Table 5-4: Pore pressure and deformation of dam. Full pore pressure is assumed in the base area without compression.*

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