

# Justification for selection of a factor of safety for dams

Investigating study of how assumptions affect the overall dam safety

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## ABSTRACT:

This paper presents a study carried out to identify how different variables affect the estimated safety of dams. To do so, a series of calculations has been carried out to understand how the factor of safety is affected for a wide range of variables and assumptions.

The calculations have been carried out by combining a computing tool for stability calculations with a script that runs the calculations. This method has proved to produce a very powerful and flexible tool for computing stability with varying assumptions. In total, the report is based on approximately 7000 separate calculations with different variables

The study gives suggestions on how uncertainties related to loads and other assumptions can be represented in the overall factor of safety. The results can also be used to justify the current practice and safety level applied by the Norwegian dam safety regulations.

This analysis is part of a larger Norwegian Research and Development project, named “Dam safety in an overall perspective” that is administrated by EnergiNorge. This is a joint project with participants from the Norwegian dam safety sector. One of the objects of this project is to look at alternative approaches to evaluate the safety of existing concrete- and masonry dams.

The study has been carried out with very limited resources and there is a need to verify the results and methodology used. We are therefore grateful for all comments related to this article and the study in general.

## 1 Introduction

Requirements for stability of concrete dams in the current Norwegian dam safety regulations are based on simplifications, which in many cases are conservative. As a result, rehabilitation works may be carried out on dams that are safe, but does not meet the safety requirements.

The factor of safety to estimate stability against sliding and overturning for dams includes many variables. How these variables affect the factor of safety are not necessarily known or accessible. It is therefore desirable understand in what way the different assumptions affect the calculated stability, in order to provide a better knowledge of the general safety level.

How different parameters affect the dam stability is also essential when assessing the degree of uncertainty of the calculations. This will help to identify which parameters that are most important for stability and sensitivity of the overall dam safety. The calculations in this study is intended as a contribution to improve the knowledge for assessing the actual safety-level for existing dams in Norway.

The calculations has been carried out on both concrete gravity dams and masonry dams. Separate calculations including ice pressure and stabilizing effect of rock bolts has also been

carried out on dam sections with a height of < 7 m. For simplicity, this paper only presents the results related to concrete gravity dams with a height > 8 m.

The study has been carried out with very limited resources and there is a need to verify the results and methodology used. We are therefore grateful for all comments related to this article and the study in general.

## 2 Requirements for dam safety in Norway

In Norway, dam stability is checked for both overturning and sliding. When calculating the sliding capacity, the current Norwegian regulations state that a plane interface between rock and the dam is to be assumed. The slope of the foundation is determined by the height difference between the dam heel and the dam toe. A friction angle between 40° and 50° can be assumed (generally 45°) and cohesion is neglected. In addition, contributions from rock bolts are neglected for dams higher than 7 m, because of uncertainties in the actual capacity and the general condition of the bolts.

Calculation of the sliding resistance require a safety factor of minimum 1.5 against normal design loads. For accident loads a minimum factor of safety of 1.1 is applied.

The above-mentioned factors of safety against sliding apply when cohesion is not included. If cohesion is included, a higher factor of safety is defined. However, as there are no easily applicable methods for identifying cohesion, this is generally not included in calculations of sliding stability.

Safety against sliding is estimated with the shear friction factor method, where the factor of safety is generally defined as the following:

$$SF_{sliding} = \frac{\sum F_{horizontal\ capacity}}{\sum H_{horizontal\ load}} = \frac{\sum V \tan(\phi + \alpha)}{\sum H}$$

where  $\phi$  is the fiction angle and  $\alpha$  is the inclination of the foundation.

When calculating the stability against overturning, the dam is assumed infinitely rigid. The resultant force is required to be within the central dam foundation so that it can be assumed pressure throughout the dam foundation.

## 3 Assumptions for calculations

### 3.1 Methodology

The calculations are based on computing tool for stability control, developed by Dr. Techn. Olav Olsen. To make the calculations more efficient, a script has been developed with the software Python, which is a widely used programming language for scientific use. The script defines changes of different variables, and then automatically generates the calculations for stability with these assumptions.

The method has provided a very powerful and flexible tool for estimating stability of all types of concrete dams with different variables. In total, the report is based on approximately 7000 separate calculations with different variables.

The result of the stability calculation of each parameter is presented graphically in the original report, where the resulting factor of safety is plotted against the varying parameters for each dam height. Variation in the factor of safety are shown for both sliding and overturning.

To simplify the output of safety against overturning, the factor of safety is calculated instead of the eccentricity of the resultant force (which is the stability criteria of the Norwegian dam safety regulations). Safety against overturning is therefore defined as:

$$SF_{overturning} = \frac{\sum M_{stab.}}{\sum M_{destab.}}$$

**3.2 Variables**

Assumptions of for the calculations are shown in the table below. “Initial values” are used to generate dam section as described in the next chapter,

Table 1. Assumptions used for the computations

Variable	Initial value	Minimum value	Maximum value	Step for variation	Comment
<b>Friction angle</b>	40°	35°	60°	1°	
<b>Water level (H<sub>w</sub>)</b>	h	h – 1 m	h	0.01 m	h = Dam height
<b>Self-weight:</b>	22 kN/m <sup>3</sup>	21 kN/m <sup>3</sup>	24 kN/m <sup>3</sup>	0.1 kN/m <sup>3</sup>	
<b>Drainage constant (k)**</b>	1.00	0.50	1.0	0.05	Changes in pore pressure are calculated by varying k and dx**.
<b>Drainage position (dx)**</b>	0	0.1H <sub>w</sub>	0.5H <sub>w</sub>	0.1H <sub>w</sub>	

\* Rock bolts, diameter 25 mm<sup>2</sup> and capacity 180 N/mm<sup>2</sup>  
 \*\* Both the drainage constant (k) and the drainage position (dx) was changed, as illustrated in the figure below (i.e. resulting in 6 \* 11 = 66 different pore pressures for each different dam height)

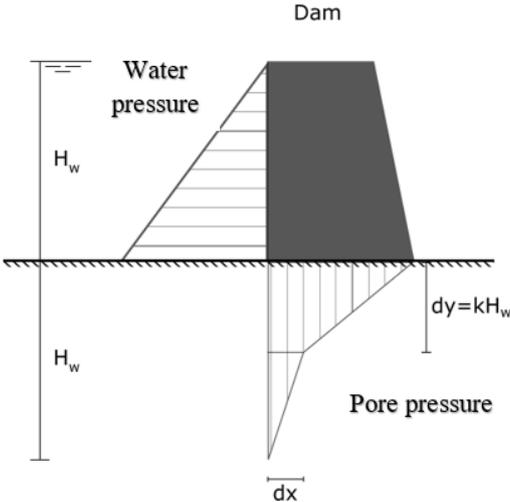


Figure 1. Illustration of assumptions to generate pore pressure.

### 3.3 Generation of dam section

For each dam height a cross section is generated that satisfies the following requirements:

- Pressure throughout the entire foundation (i.e. linear decreasing pore pressure throughout the interface between dam and foundation)
- Factor of Safety against sliding equal to 1.0. The Factor of safety is increased if a suitable cross section is not found.

By varying crest width and downstream slope (see Figure 2) an optimal cross section is found using the “initial values” given in Table 1.

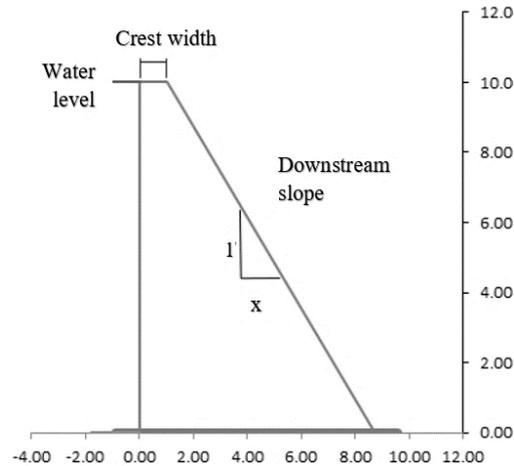


Figure 2. Dam section is selected by varying crest width and downstream slope.

It was not possible to generate a cross section that satisfied the assumptions mentioned above. Therefore, the required limit for “factor of safety” was increased from 1.0 to 1.1 as shown in the table below. To compare the correlation between load and factor of safety for different dam heights, the factor of safety was normalized so that it equals to 1.0.

Table 2. Geometric values and safety factor for dam sections generated.

Dam height [m]	Crest width [m]	Downstream slope [1:x]	Factor of Safety	
			Sliding	Overtuning
8	0.81	0.77	1.1	1.5
10	1.01	0.77	1.1	1.5
12	1.21	0.77	1.1	1.5
14	1.41	0.77	1.1	1.5
16	1.61	0.77	1.1	1.5
18	1.82	0.77	1.1	1.5
20	2.02	0.77	1.1	1.5
25	2.52	0.77	1.1	1.5
30	3.03	0.77	1.1	1.5

The above table shows that optimization of the cross sections provided a minimum factor of safety of 1.1 against sliding and 1.5 against overturning, with the assumptions used. Normalization of the results imply that the computed results for factor of safety against sliding is divided by 1.1, while the results against overturning is divided by 1.5.

It can also be noted that when the friction angle is increased from 40° to 50°, the safety factor against sliding will be about 1.5. i.e. the same as the factor of safety against overturning.

### 4 Results

In this chapter, the results of the calculations with different variables are given and discussed.

#### 4.1 Friction angle (and angle of foundation)

Variation in friction angle is also valid for inclination of the foundation. E.g. a foundation inclination of 5° to the downstream side will reduce the friction angle with 5°, while an inclination to the upstream side will increase the friction angle with 5°.

The friction angel has no effect on the factor of safety against overturning. When it comes to sliding, the computations show that variations in the friction angle is directly related to the factor of safety against sliding. The dam height does not influence the correlation between factor of safety and friction angle.

The relationship between friction angle and factor of safety against sliding is shown in the figure below. The graphs have approximately the same curvature, so that changes in the friction angle have approximately the same effect on the factor of safety regardless of the original value. In other words, when the design friction angle is 5° higher than the actual friction angle, this will result in a factor of safety of 1.20, regardless of the original design value.

The Norwegian guideline for concrete dams, allow a friction angle between 40° and 50°, dependent on the rock quality. Cohesion is generally not included in the sliding resistance. The friction angle will normally be conservative where the friction angle will also cover possible cohesion and shear capacity due to rock surface roughness.

The calculations in the report show that a conservative friction angle will give a high level of safety that is not necessarily reflected in the computed factor of safety for the dam.

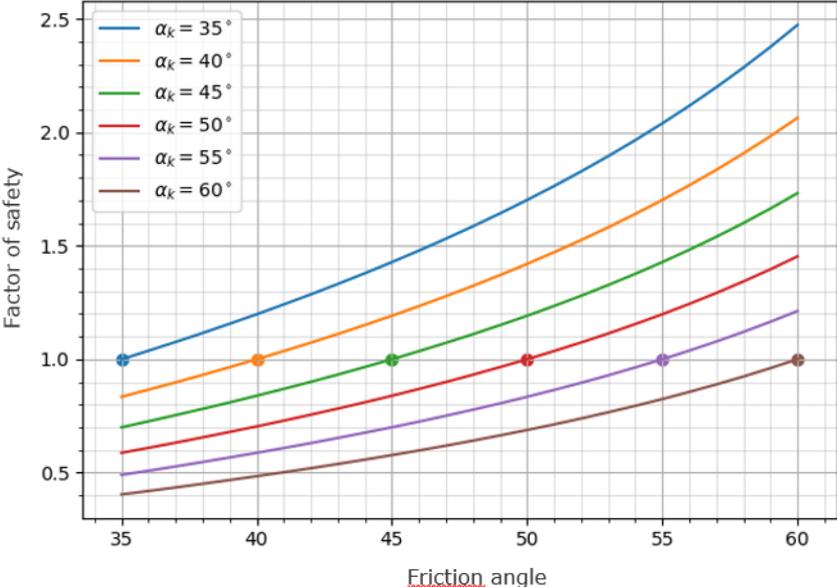


Figure 3. Correlation between factor of safety and friction angle.  $\alpha_k$  is the initial friction angle where the dam cross-section have factor of safety = 1.0.

## 4.2 Water level

How stability is affected by changes in the water level identifies how sensitive the dam will be to changes in flood water level. Changes in design water level can for instance be caused by changes in future flood calculations etc. How much this affects the safety in relation to different dam heights is calculated and presented graphically in the following figures.

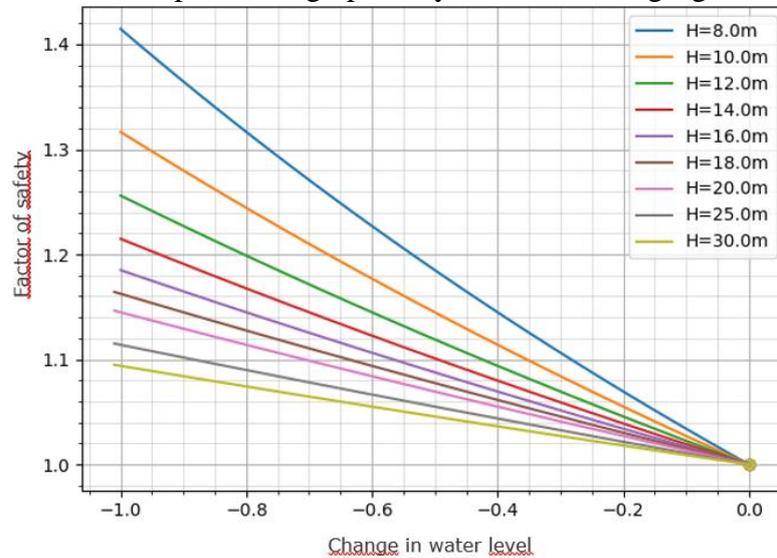


Figure 4. **Sliding:** Reduced water level vs. factor of safety for different dam heights.

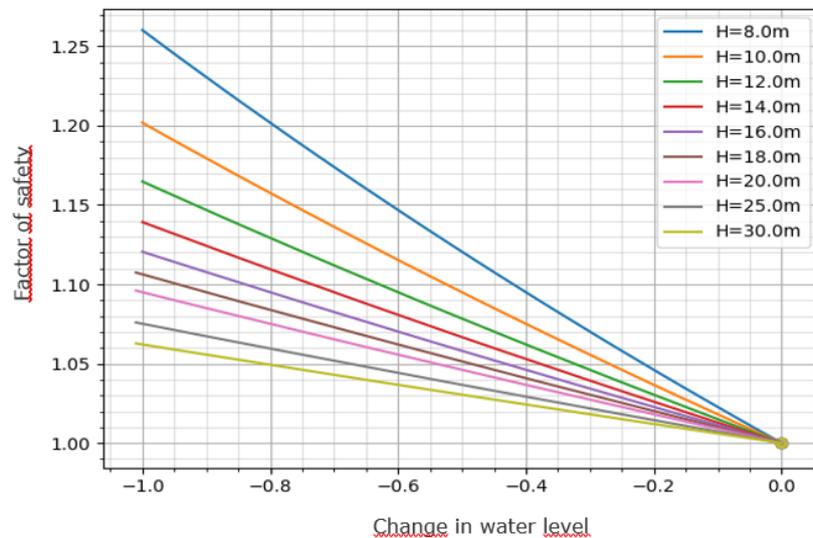


Figure 5. **Overturning:** Reduced water level vs. factor of safety for different dam heights.

As shown in the above graphs, higher dams are of course less sensitive to changes in water levels than lower dams. This is summarized in the following table.

Change in water level	Sliding Factor of safety		Overturning Factor of safety	
	Dam height 8 m	Dam height 30 m	Dam height 8 m	Dam height 30 m
0,2 m	1,07	1,02	1,05	1,01
1,0 m	1,41	1,09	1,26	1,06

Figure 6. Effects of changes in water level on the factor of safety for different dam heights.

The table shows that changes in water levels are more important for safety against sliding than against overturning.

Dam height (i.e. static water pressure) is crucial for how uncertainties in flood calculation and flooding affect stability. When the dam height increases, changes in flood water have little significance for the dam stability.

As uncertainties in floods and operating levels will have different impact on the factor of safety dependent on the dam height, it is reasonable that these uncertainties are handled in the flood calculations and are not included in the factor of safety. For instance, a dam dependent on flood gates will have other uncertainties related to flood handling and flood levels than a dam with a free overflow spillway.

**4.3 Self-weight**

The self-weight is, of course, essential for the stability of a concrete gravity dam. The calculations carried out show that variations in the self-weight is directly related to the factor of safety. The dam height does not influence the correlation between factor of safety and self-weight.

If the self-weight is reduced from 24 to 23 kN/m<sup>3</sup>, and represents a load factor of 0.96. This corresponds to a coefficient of variation of 0.04 as recommended by JCSS, "Probabilistic Model Code," Joint Committee on Structural Safety, 2015, Table 2.1.1. Another reference could be the Eurocode where a load factor of 0.9 can be assumed for self-weight with a stabilizing effect, to take account of uncertainties in geometry and self-weight. The correlation between the factor of safety and load factor is shown in the table below.

Table 3. Correlation between load factor and factor of safety.

	Load factor	Factor of safety
<b>Sliding</b>	0,96	1,08
<b>Overturning</b>	0,96	1,05

A graphic presentation of the correlation between self-weight and factor of safety is shown below.

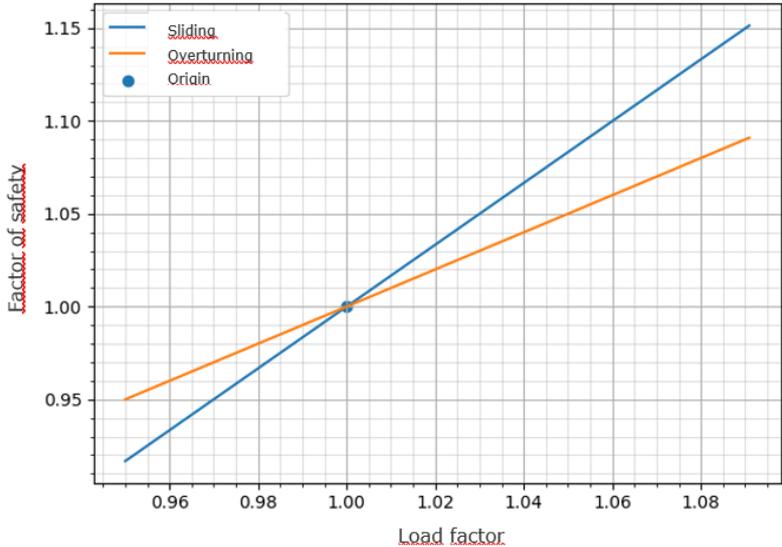


Figure 7. Load factor vs. factor of safety (Friction angle = 40°)

The dam geometry also represents an uncertainty, but how this affects the factor of safety has not been investigated. Probabilistic analysis carried out on Dam Reinoksvatn, indicate however that deviations in the geometry do not have a significant effect on the factor of safety. This analysis is presented in the workshop as an article under the “open theme”.

**4.4 Pore pressure**

The pore pressure represents an uncertainty that can be difficult to predict and therefore difficult to quantify in terms of a specific factor of safety. This would imply that the pore pressure should be subjected to a relatively high factor of safety to take account of the uncertainty it represents.

In Norway resultant force should act upstream center third of the foundation when the dam is subjected to normal design loads. Thereby, a linear decreasing pore pressure can be assumed as there is pressure throughout the entire dam foundation. In addition, a check of accident load is required, where the resultant force should be upstream 1/6 of the dam foundation. In this case, full pore pressure can be assumed on the upstream half of the foundation (where there is no pressure on the foundation) and then linearly decreasing to the downstream side. The assumptions for design loads and accident loads are shown in the following figure.

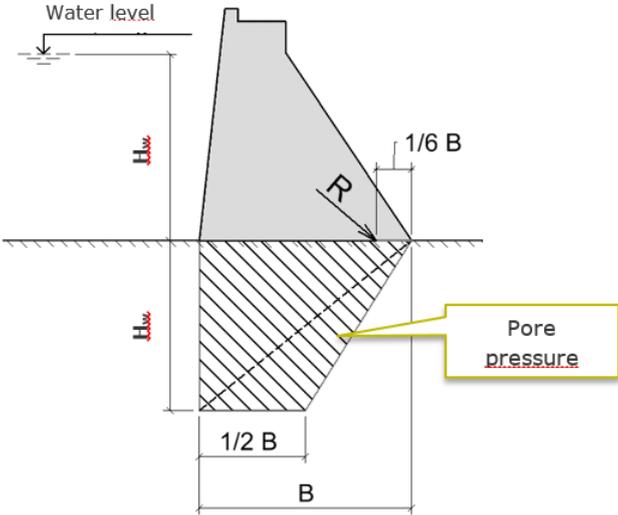


Figure 8. Maximum allowable pore pressure assumed for accident loads. Pore pressure distribution for normal design loads is shown as a dotted line.

The criteria for pore pressure distribution provides a logical correlation between the load effects from the dam and the pore pressure for normal design loads. When there is pressure in the entire foundation, the bond between the concrete and the foundation can be assumed to be intact. Thereby, a linearly decreasing pore pressure under the dam will probably be a conservative assumption and generally contribute to a high safety level.

The additional check for accident loads provides an extra safety in case the pore pressure should be greater than assumed for normal design loads.

If the maximum permissible pore pressure for accident loads represents the uncertainty in the pore pressure distribution, the difference of pore pressure between design load situation and accident load situation may be looked upon as the corresponding load factor. This difference represents an increase in pore pressures of 43%, i.e. a load factor of 1.43. The correlation between the factor of safety for design loads and accident loads can thus be expressed as shown in the following table and figure.

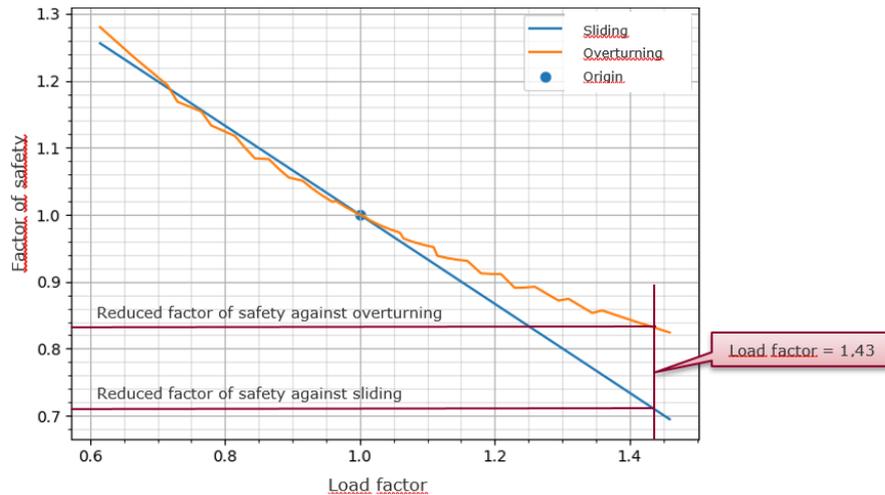


Figure 9. Computed correlation between load factor and factor of safety with changing pore pressure. Origin represent a linear pore pressure from upstream to downstream side.

Load factor is taken as the difference in allowable pore pressure between accident loads and normal design loads. From the graph, the correlation between pore pressure and factor of safety is summarized in the following table. The dam height does not influence the correlation between factor of safety and pore pressure.

Table 4. Pore pressure: Correlation between load factor and factor of safety.

	Load factor	Factor of safety		Comment
		Accident Loads	Design loads	
<b>Sliding</b>	1.43	1.0	1.41	=1/0,71 see Figure 9
<b>Overturning</b>	1.43	1.0	1.20	=1/0,83 see Figure 9

## 5 Summary and conclusion

The following table summarizes the suggested factor of safety for each variable as discussed in this paper.

Multiplying the different factors is assumed to represent the overall factor of safety.

Variable	Factor of safety				Comments
	Sliding		Overturning		
	Design loads	Accident loads	Design loads	Accident loads	
<b>Friction</b>	1,0	1,0	Not relevant		Safety is accounted for by conservative values for friction angle.
<b>Water level</b>	1,0	1,0	1,0	1,0	Uncertainties in design water level should be reflected in the flood calculation.
<b>Self-weight</b>	1,08	1,08	1,05	1,05	Suggested factor of safety represent a load factor of 0.96.
<b>Pore pressure</b>	1,40	1,00	1,20	1,00	The factor of safety corresponds to the difference in safety between the maximum acceptable pore pressure for design loads and accident loads.

Variable	Factor of safety				Comments
	Sliding		Overturning		
	Design loads	Accident loads	Design loads	Accident loads	
<b>SUM</b>	<b>1,51</b>	<b>1,08</b>	<b>1,26</b>	<b>1,05</b>	<b>= suggested total factor of safety (= factors for all variables multiplied together)</b>
<b>Current requirements</b>	<b>1,5</b>	<b>1,1</b>	<b>N.A.*</b>	<b>N.A.*</b>	<b>* Safety against overturning is defined by position of the resultant</b>

Elements constituting the total factor of safety given in the Norwegian dam safety regulations is not publicly available. The factors of safety suggested in the above table can, however, be used to justify the current requirements, but this has not been confirmed or commented by the Norwegian dam safety authority.

How different parameters affect the dam stability is essential when assessing the degree of uncertainty of the calculations. This will make it easier to identify which parameters that are most important for the stability and that influences the sensitivity of the overall dam safety. This is of interest particularly in cases where existing dams do not meet the safety requirements. By improving the knowledge related to the individual variables the uncertainties can be reduced and thereby reducing the overall required factor of safety for the dam in question.

It must be underlined that results in this report is valid with the given methodology and assumptions described in chapter 3.1 and 3, and a validation of the results is recommended.

## 6 Acknowledgements

We would like to thank EnergiNorge that has supported this project, and thereby made this contribution possible.

## 7 References

This article is based on a study documented in a report by EnergiNorge in Norwegian. When this article was written, the report was not published. The report will in time available on: <https://www.energinorge.no/publikasjoner/>