Theme D – Risk analysis – assessment of reliability for concrete dams

Submission for workshop D by Dr.techn.Olav Olsen AS

Hovde E.[1], Strand M.[1], Konow T.[1] and Engseth M.[1]

[1]*Dr.techn.Olav Olsen, Oslo, Norway*

E-mail: thk@olavolsen.no

ABSTRACT:

As a part of the 14th International Benchmark Workshop in Numerical Analysis of Dams organized by ICOLD Committee on Computational Aspects of Analysis and Design of dams, this paper gives a summary of the calculations, assumptions and results of Dr.techn.Olav Olsen AS for theme D – Risk Analysis – assessment of the reliability for concrete dams.

The analysis is conducted with SOFiSTiK module Rely. SOFiSTiK is a program mainly used for FEM-analysis and structural design. Rely is an add-on to the SOFiSTiK program that performs reliability analysis, where the engineering system of interest is modeled using one of the SOFiSTiK finite element modules. The kernel of Rely is powered by the stand-alone software package Strurel.

1 Introduction

This paper is a part of the 14th International Benchmark Workshop in Numerical Analysis of Dams organized by ICOLD Committee on Computational Aspects of Analysis and Design of dams. The paper gives a summary of the calculations, assumptions and results of Dr.techn.Olav Olsen AS for theme D – Risk Analysis – assessment of the reliability for concrete dams.

The analysis is conducted with SOFiSTiK module Rely. SOFiSTiK is a program mainly used for FEM-analysis and structural design. Rely is an add-on to the SOFiSTiK program that performs reliability analysis, where the engineering system of interest is modeled using one of the SOFiSTiK finite element modules. The kernel of Rely is powered by the stand-alone software package Strurel [1]. The concrete dam has not been modeled in finite elements, but the stability calculations is coded with the input language CADINP for a free input format.

The paper includes a chapter for each task in the problem description for this theme.

2 Deterministic factor of safety – task 1

2.1 Assumptions

In the problem description, the dam height is given as 25m, but the drawing in appendix 1 shows that it is 0.6 meters taller. The concrete-rock contact where failure is assumed is taken as elevation +226.9 m after what is shown on the attached drawing. This gives a total dam height of 25.6 m and a retention water level of 24.1 m at elevation +251.0 m. The water level at the toe is assumed to be as 3.1 m above the foundation for both cases.

When calculating the resistance against failure for the rock joint, the inclination of the joint itself is added to the friction and dilatation angles.

The uplift pressure is assumed to be linearly decreasing from upstream water level at the heel to the water level at the downstream toe of the dam, if the resultant is within 1/3 of the base width. If the resultant is downstream this area, the uplift pressure is assumed equal to the water pressure at the upstream heel over the area without compression. The resultant and uplift pressure is iterated until equilibrium is achieved.

The hydrostatic pressure on the rock segment above the rock joint is calculated from elevation +217.9 m (+226.9m-9m), on both the upstream and downstream side. At the toe the uplift pressure is then 3.1 m due to the inclination of the rock joint. At the rock joint, the pore pressure is therefore decomposed in a vertical and horizontal component.

The table below shows a summary of values used to calculate the deterministic factor of safety. Values marked * are later used as variables in the probabilistic analysis.

Table 1: Values for deterministic calculations

Retention water level	24.1 m
Downstream water level	3.1 m
Water density	9.81 kN/m^3
Concrete self-weight*	24 kN/m^3
Rock self-weight*	26 kN/m^3
Ice load*	200 kN/m
Friction angle: Concrete-rock*	35°
Dilatation angle: Concrete-rock*	15°
Friction angle: Rock joint*	32°
Dilatation angle: Rock joint*	8°
Inclination of the rock joint	20°
Jacking loss*	10%
Flood level*	1.5 m

2.2 Results

The deterministic factors are presented in the table below.

Table 2: Deterministic factor of safety for sliding

	Concrete-rock	Rock joint
Normal load case	1.864	1.825
Flood load case	1.714	1.660

3 Limit state functions – task 2

The limit state functions are given as:

$$R - F > 0 \tag{1}$$

Where:

R is resistance
F is loading

Failure happens when the limit state function is less than, or equal to, zero. To achieve convergence the software must also know how close it is to failure.

3.1 Sliding along the concrete-rock contact

In the analysis, the resistance for sliding along the concrete-rock is defined as:

$$R = N' \cdot \tan(\varphi + i) \tag{2}$$

Where:

N’ is the sum of vertical forces
φ is the friction angle
i is the dilatation angle

The vertical forces, N’, include; the self-weight of concrete, jacking force and pore pressure.

The loading, F, which is the sum of all horizontal forces, include; Water pressure (both upstream and downstream) and ice load (for the normal load case).

3.2 Sliding along the rock joint in the foundation

In the analysis, the resistance for sliding along the rock joint is defined as:

$$R = N' \cdot \tan(\varphi + i + a) \tag{3}$$

Where:

N’ is the sum of vertical forces
φ is the friction angle
i is the dilatation angle
a is the inclination of the rock joint

The vertical forces, N’, include; the self-weight of concrete, self-weight of the rock above the rock joint and vertical component of the pore pressure along the rock joint.

The loading, F, which is the sum of all horizontal forces, include; Water pressure (both upstream and downstream) and ice load (for the normal load case).

4 Estimation of probability of failure – task 3

4.1 Definition of variables

The variables describing the loads acting on the dam and the resistance of the structure is modeled with probability density functions (PDF). A common PDF for natural random variables is a normal distribution, described by a mean value and a standard deviation. Other distributions used in this project are log-normal distributions, which constrain the PDF to only positive values and trapezoid distributions. The distributions implemented here are taken from the problem description or [2]. Table 3 show a summary of the distributions. The variables where assumptions are made, are explained in depth in the following text and marked with * in the table.

The water level downstream is kept constant for all cases.

Table 3: Probability distributions

	Distribution type	Mean value	Coefficient of variation	Standard deviation
Concrete self-weight*	Normal	24 kN/m^3	0.04·0.85	0.816 kN/ m^3
Rock self-weight	Normal	26 kN/m^3	0.02	0.53 kN/m^3
Ice load*	Trapezoid	112 kN/m	0.6	67 kN/m
Friction angle: Concrete-rock	Normal	35°	0.05	1.75°
Dilatation angle: Concrete-rock	Log-normal	15°	0.2	3°
Friction angle: Rock joint	Normal	32°	0.07	2.24°
Dilatation angle: Rock joint	Log-normal	8°	0.4	3.2°
Jacking loss	Normal	10%	0.3	3%
Flood level*	Trapezoid	1.03 m	0.74	0.766

Concrete self-weight

The concrete self-weight is taken as 24 kN/m^3, which is the middle value of the two values listed in table PIII 2-1. [2]

Ice load

The dam is located in the north of Sweden, which gives a maximum ice thickness of 1 meter. The probability distribution should be as given for Norrland, according to table PII 2-1 [2]. The ice load distribution is to be truncated to a maximum value of 250 kN/m, due to buckling of the ice itself. In PMCD, the truncation is given with a standard deviation of 25 kN/m.

A method for truncation was not found in the manual for the SOFiSTiK Rely-module. Not implementing the truncation, leads to an unrealistically high ice load and a high probability of failure for the normal load case. We therefore assumed an ice load distribution with a trapezoidal distribution with a maximum ice load value of 300 kN/m, the mean value of the truncation plus two standard deviations. The figure below show a comparison of the two distributions.

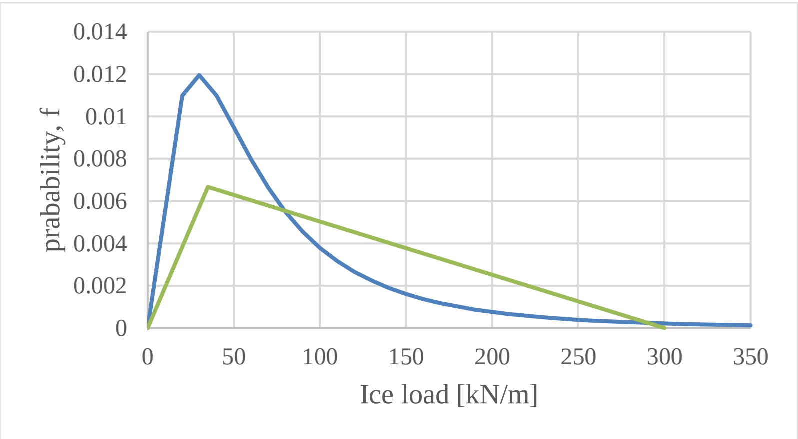

Figure 1: Comparison of ice load distributions. **Blue line:** PMCD, **Green line:** Our assumption.

Flood level

The distribution of the flood level is given by tabulated values of an original CDF. Three trapezoid distributions are given as approximations to three subintervals of the original distribution. The values outside the corresponding subinterval are supposed to be truncated. Using the SOFiSTiK Rely-module there was not found a suitable method of truncating the values outside the subintervals. Instead, one trapezoid distribution is used over the entire interval. The chosen trapezoid has parameters a=-0.1, b=0.0, c=3.15 and is shown in

Error! Reference source not found. compared with the original CDF and the three recommended trapezoid distributions. The line fit in general well to the lines of the other trapezoid lines.

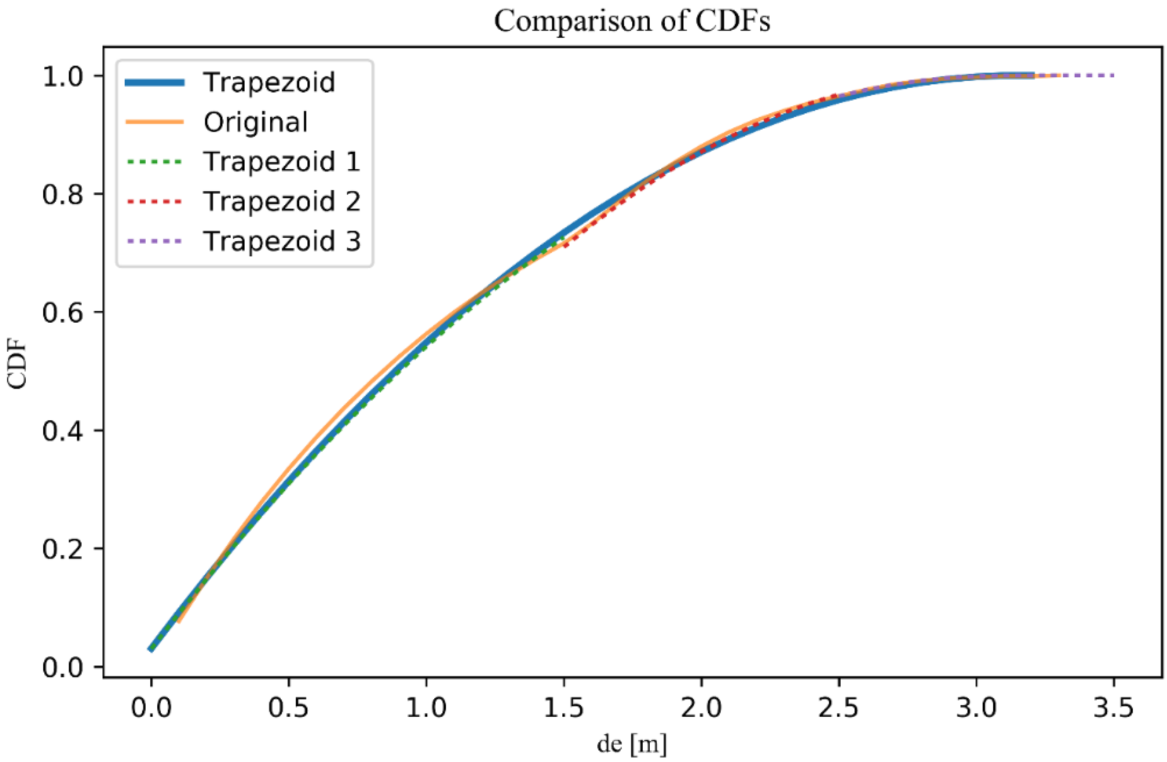

Figure 2: Comparison of CDF distributions of the water level above rwl.

The flood situation is calculated in two separate subcases: one for water height at retention water level (rwl) and one for water height above retention water level.

$LC_{ii,1}: H_w = rwl \qquad P(LCi_{ii,1}) = 1 - P(LC_{ii}) = 0.997$

$LC_{ii,2}: H_w > rwl \qquad P(LC_{ii,2}) = 3 \cdot 10^{-3}$

The two subcases can not occur at the same time and are thus mutually exclusive.

The probabilities of each failure event in the flood situation is the total probability of failure due to subcase 1 and failure due to subcase 2:

$$P(F_x) = P(F_x|LC_{ii,1}) \cdot P(LC_{ii,1}) + P(F_x|LC_{ii,2}) \cdot P(LC_{ii,2}), \qquad x = a, b \tag{4}$$

4.2 Results

Table 4: Probability and safety index, β

	Concrete-rock (F_a)		Rock joint (F_b)	
	Probability	**β**	**Probability**	**β**
Normal load case (LC_i)	$1.971 \cdot 10^{-9}$	5.89	$9.147 \cdot 10^{-8}$	5.22
Flood load case (LC_{ii}):				
Flood load case ($LC_{ii,1}$)	$4.226 \cdot 10^{-11}$	6.49	$1.708 \cdot 10^{-8}$	5.52
Flood load case ($LC_{ii,2}$)	$2.270 \cdot 10^{-4}$	3.51	$1.269 \cdot 10^{-4}$	3.66
SUM Flood case, LC_{ii}	$6.811 \cdot 10^{-7}$	4.83	$3.977 \cdot 10^{-7}$	4.93

5 Sensitivity values – task 4

Table 5 shows the sensitivity values for the analysis normal load case and the two analysis used to calculate failure of probability of the flood.

Higher sensitivity value indicates greater importance of the variable in question. Negative sensitivity indicates that the variable acts as a load and a positive sensitivity indicates resistance. [2]

Table 5: Sensitivity values

	Concrete-rock (F_a)			Rock joint (F_b)		
	LC_i	**$LC_{ii,1}$**	**$LC_{ii,2}$**	**LC_i**	**$LC_{ii,1}$**	**$LC_{ii,2}$**
Concrete self-weight	0.47	0.49	0.59	0.41	0.43	0.35
Rock self-weight	-	-	-	0.13	0.13	0.11
Ice load	-0.24	-	-	-0.18	-	-
Friction angle: Concrete-rock	0.64	0.67	0.27	-	-	-
Dilatation angle: Concrete-rock	0.55	0.55	0.34	-	-	-
Friction angle: Rock joint	-	-	-	0.78	0.79	0.65
Dilatation angle: Rock joint	-	-	-	0.42	0.42	0.44
Jacking loss	-0.08	-0.08	-0.13	-	-	-
Flood level	-	-	-0.66	-	-	-0.49

6 System reliability of the monolith – task 5

The reliability of failure of the monolith is calculated for two failure modes separately due to two different design situations, leading to four component reliability values. The system reliability of the monolith is calculated based on a non-redundant system where failure in one component leads to failure in the entire construction. Non-redundant systems are analyzed as series systems. [2]

The component failure events are:

F_a: sliding along the concrete-rock contact

F_b: sliding along a rock joint in the foundation

The failure events are considered independent of each other, thus the joint probability of the events is:

$$P(F_a \cap F_b) = P(F_a) \cdot P(F_b) \quad (5)$$

The failure events are conditional on two load cases:

LC_i: normal design situation

LC_{ii}: flood situation

For a series, the system failure event is the union of the two component failure events, such that the probability of failure for a given load case is

$$P_f = P(F_a \cup F_b) = P(F_a) + P(F_b) - P(F_a \cap F_b) \quad (6)$$

The system reliability index β is defined as

$$\beta = -\Phi^{-1}(P_f) \quad (7)$$

Φ is the standard normal cumulative distribution function and Φ^{-1} is its inverse function.

Table 6: Probabilities of failure and system reliability for the two load cases.

	LC_i	**LC_{ii}**
$P(F_a)$	$1.971\cdot10^{-9}$	$6.811\cdot10^{-7}$
$P(F_b)$	$9.147\cdot10^{-8}$	$3.977\cdot10^{-7}$
P_f	$9.345\cdot10^{-8}$	$1.079\cdot10^{-6}$
β	5.21	4.73

7 Effect of shear test – task 6

The basic friction angle ϕ_b of the concrete-rock contact was assumed to have a given distribution of the mean value. With two test samples the prior knowledge of the mean can be updated according to section 8 part I in [2]. The information about the variables are summarized in Table 7.

The mean of the friction angle was given as a normally distributed variable $\mu'_{\phi_b} \sim \mathcal{N}(35°, 1.75°)$. The friction angle parameter has an on-site variability $V_{\phi_b} = 0.03$, such that the standard deviation of the parameter is $\sigma_{\phi_b} = V_{\phi_b} \cdot E\left[\mu'_{\phi_b}\right] = 1.05°$. The primary friction angle is given by the linked distribution $\phi_b \sim \mathcal{N}(\mu'_{\phi_b}, 1.05°)$.

The distribution of the mean can be updated Bayesian inference. By using equations ([2] eq 6 and 7) the updated expectance $E\left[\mu''_{\phi_b}\right]$ and variance $Var\left[\mu''_{\phi_b}\right]$ are calculated based on the a priori assumption and the test results. By assuming no measurement error and no spatial correlation, the total variability on the mean may be approximated as $V\left[\mu_{\phi_b}\right] = \sqrt{V^2_{\phi_b} + V^2_{stat,\mu_{\phi_b}}}$, where $V^2_{stat,\mu_{\phi_b}} = \frac{Var\left[\mu''_{\phi_b}\right]}{E\left[\mu''_{\phi_b}\right]^2}$.

The updated mean is given by $\mu''_{\phi_b} \sim \mathcal{N}(37.1°, 1.25°)$ and the updated friction angle is given by the linked distribution $\phi_b \sim \mathcal{N}(\mu''_{\phi_b}, 1.11°)$. Compared to the a priori assumption the updated mean is higher, which leads to a more stable dam and the variability is lower.

Table 7: Distributions for basic friction angle for first and updated assumption on mean value.

Description	**Variables**	
	First assumption	
Mean: expectation and variance	$E[\mu'_{\phi_b}] = 35°$	$Var[\mu'_{\phi_b}] = (1.75°)^2 = 3.06$
On-site variability	$V_{\phi_b} = 0.03$	
Standard deviation of basic friction angle	$\sigma_{\phi_b} = V_{\phi_b} \cdot E[\mu'_{\phi_b}] = 1.05°$	
Distributions	$\phi_b \sim \mathcal{N}(\mu'_{\phi_b}, 1.05°)$	$\mu'_{\phi_b} \sim \mathcal{N}(35°, 1.75°)$
	Updated assumption	
Test samples	$\phi_{b,1} = 37°$	$\phi_{b,2} = 38°$
Sample mean and number	$m_{\phi_b} = 37.5°$	$n_{\phi_b} = 2$
Mean: expectation and variance	$E[\mu''_{\phi_b}] = 37.1°$	$Var[\mu''_{\phi_b}] = 0.47$
Total uncertainty of mean	$V[\mu_{\phi_b}] = 0.034$	$\sigma[\mu''_{\phi_b}] = V[\mu_{\phi_b}] \cdot [\mu''_{\phi_b}] = 1.25°$
Standard deviation of basic friction angle	$\sigma_{\phi_b} = V_{\phi_b} \cdot E[\mu''_{\phi_b}] = 1.11°$	
Distributions	$\phi_b \sim \mathcal{N}(\mu''_{\phi_b}, 1.11°)$	$\mu''_{\phi_b} \sim \mathcal{N}(37.1°, 1.25°)$

$$E[\mu''_{\phi_b}] = \frac{m_{\phi_b} Var\left[\mu'_{\phi_b}\right] + E\left[\mu'_{\phi_b}\right]\frac{\sigma^2_{\phi_b}}{n_{\phi_b}}}{Var\left[\mu'_{\phi_b}\right] + \frac{\sigma^2_{\phi_b}}{n_{\phi_b}}} \tag{8}$$

$$Var[\mu''_{\phi_b}] = \frac{Var\left[\mu'_{\phi_b}\right]\frac{\sigma^2_{\phi_b}}{n_{\phi_b}}}{Var\left[\mu'_{\phi_b}\right] + \frac{\sigma^2_{\phi_b}}{n_{\phi_b}}} \tag{9}$$

The table below show probability for the concrete-rock normal load case with updated friction angle and the mean as its own variable. The failure probability is reduced from $1.866 \cdot 10^{-8}$ to $5.902 \cdot 10^{-12}$.

Table 8: Change in the probability due to updated friction angle

	LC_i	**LC_i – with updated angles**
P_f	$1.866 \cdot 10^{-8}$	$5.902 \cdot 10^{-12}$
β	5.50	6.78

Due to limitations in the software we were unable to insert the mean as its own variable as shown in the table and get sensitivity values for the variables. The sensitivity values were not requested in the problem description, but in the excel-sheet for submitting results. To find the sensitivity values a separate analysis was run with an updated basic friction angle as $\phi_b \sim \mathcal{N}(37.1°, 1.25°)$. The results are shown in the table below.

Table 9: Sensitivity values for updated friction angle

	LC_i	**LC_i – with updated angle**
P_f	$1.971 \cdot 10^{-9}$	$1.368 \cdot 10^{-11}$
β	5.89	6.66
Concrete self-weight	0.47	0.81
Ice load	-0.24	-0.35
Friction angle	0.64	0.24
Dilatation angle	0.56	0.34
Jacking loss	-0.08	-0.17

The comparison shows that the sensitivity of the friction angle decreases when updating the friction angle based on tests. With an updated friction angle, the concrete self-weight has now the highest sensitivity value.

8 Conclusions

The table below shows the estimated failure probabilities for the two load cases and failure modes. The flood load case gives the highest probability of failure for both failure modes, where the concrete-rock interface has the highest failure probability of these.

	Concrete-rock (F_a)		**Rock joint (F_b)**	
	Probability	**β**	**Probability**	**β**
Normal load case (LC_i)	$1.971 \cdot 10^{-9}$	5.89	$9.147 \cdot 10^{-8}$	5.22
Flood load case (LC_{ii})	$6.811 \cdot 10^{-7}$	4.83	$3.977 \cdot 10^{-7}$	4.93

The PMCD gives a suggestion for a target reliability index for ultimate limit states. The values from this analysis corresponds to dam consequence class B, se figure below.

Dam consequence class	Minimum β minimum
A	5,2
B	4,8
C U	4,2 3,8

Figure 3: Minimum values for β in ultimate limit states [2]

The sensitivity values presented in chapter 0 shows that the friction angle has a significant impact on the result and that preforming shear tests can be valuable to increase the safety factor, as seen in chapter 7. Additional test of the concrete density may increase the safety factor even further.

9 Acknowledgements

We would like to thank EnergiNorge that has supported this project, and thereby made this contribution possible.

10 References

[1] SOFiSTiK AG, RELY Manual, SOFiSTiK, 2016-5.

[2] M. W. Wilde and F. Johansson, "Probabilistic model code for concrete dams," Energiforsk AB, 2016.